ANGLO EUROPEAN SCHOOL

IB HL Mathematics: Analysis & Approaches Bridging Work

Name: _____

Introduction

Congratulations on choosing to study IB HL Mathematics: Analysis and Approaches. This booklet contains three special GCSE papers made from only the hardest questions from the past few years, as well as their mark schemes. To help you prepare for studying IB HL Maths, complete all of the questions in these three papers to help brush up on the skills you have learned at GCSE. As a guide, students who achieve Grade 9 at GCSE would be expected to achieve approximately 50 out of 80 marks in each of these papers.

There will be a test in the first week of term. It is expected that IB HL students will demonstrate an _exceptional_ level of success.

Please bring your completed and self-marked work from this booklet to your first maths lesson in September to show your teacher.

Differences between GCSE and IB HL maths:

GCSE Maths	IB HL Maths
If you're naturally good at maths you can do well without much extra studying.	Everybody will need to do a lot of study outside of class.
It's the answer that matters most, but you should show working.	It's the method that matters, not the answer. Often you are given the answer and you need to show steps in the method.
You have an exercise book to keep all your work together.	You will need to keep neat, accurate and well-ordered notes and work in your folder.
How you present your work is not overly important, as long as you get there in the end.	How you present your work can make a big difference to whether you get the right answer at all and whether anyone can understand your method.

Additional website resources:

You may find the following websites useful for revising GCSE topics:

- Dr Frost – Click 'Practise' and choose 'Practise by Topic' (requires free sign-up if you don't yet have an account) https://www.drfrostmaths.com

- Maths Genie – GCSE questions with model solutions; also has videos. https://www.mathsgenie.co.uk/gcse.html

- Corbett Maths – GCSE questions with model solutions; also has videos. https://corbettmaths.com/contents/

- Exam solutions – videos and GCSE questions with mark schemes https://www.examsolutions.net/gcse-maths/

- Khan academy – linked to American school syllabus, but has very clear videos for many topics https://www.khanacademy.org/math/algebra-home

- Mathswatch – interactive GCSE questions and videos. If you studied with us for Year 11 you will already have an account; click 'Videos' and search for the topic you wish to study then click 'Interactive questions' https://vle.mathswatch.co.uk/vle/

Additional reading list:

You may find the following books interesting:

- Alex's Adventures in Numberland by Alex Bellos

- Cabinet of Mathematical Curiosities by Ian Stewart

- The Num8er My5teries by Marcus du Sautoy

- How Many Socks Make a Pair?: Surprisingly Interesting Maths by Rob Eastway

- The Curious Incident of the Dog in the Night-time by Mark Haddon

- The Penguin Dictionary of Curious & Interesting Numbers by David Wells

- The Calculus Wars by Jason Socrates Bardi

- The Code Book by Simon Singh

- 50 Mathematical Ideas You Really Need to Know by Tony Crilly

Candidate surname		Other names

Pearson Edexcel
Level 1/Level 2 GCSE (9–1)

Centre Number	Candidate Number

Aiming for 9 – Spring 2020 practice

Morning (Time: 1 hour 30 minutes)	Paper Reference **1MA1/1H**

Mathematics
Paper 1 (Non-Calculator)
Higher Tier

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.
Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **show all your working.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may not be used.**

Information

- The total mark for this paper is 80. There are 20 questions.
- Questions have been arranged in an ascending order of mean difficulty, as found by Grade 7 students in June and November examinations.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.
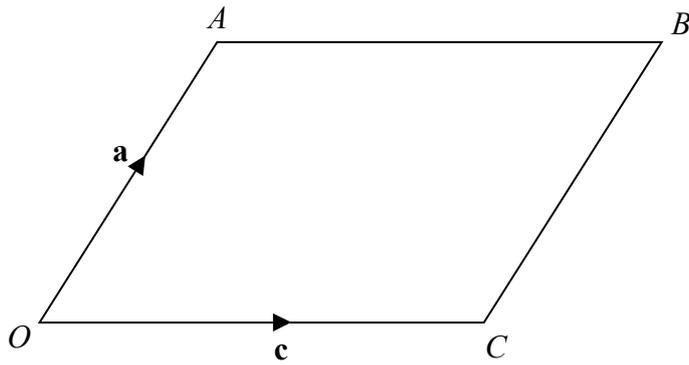
Write your answers in the spaces provided.

You must write down all the stages in your working.

1 n is an integer.

Prove algebraically that the sum of $\frac{1}{2}n(n+1)$ and $\frac{1}{2}(n+1)(n+2)$ is always a square number.

(Total for Question 1 is 2 marks)

_____

_____

2

OABC is a parallelogram.

$$\overrightarrow{OA} = \mathbf{a} \quad \text{and} \quad \overrightarrow{OC} = \mathbf{c}$$

X is the midpoint of the line *AC*.
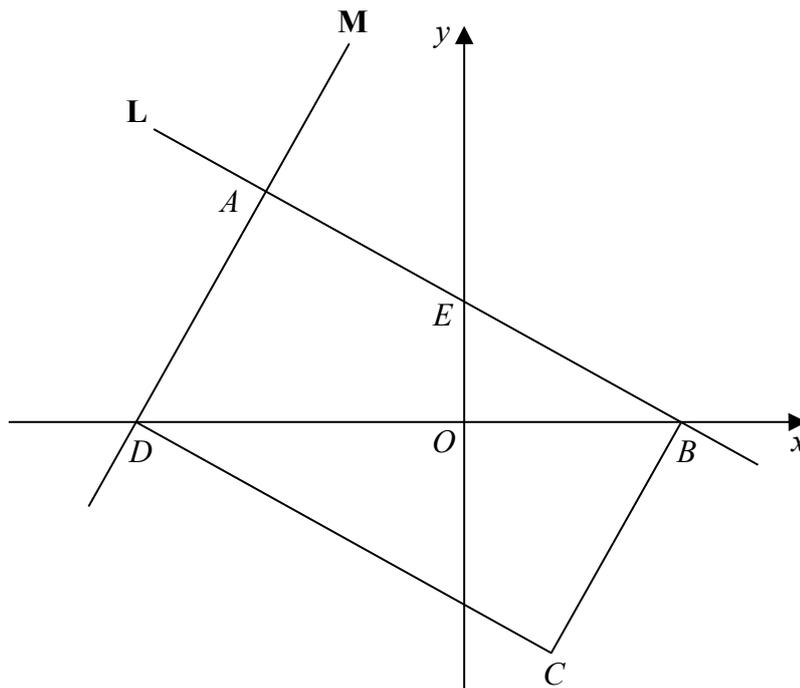*OCD* is a straight line so that $OC : CD = k : 1$

Given that $\overrightarrow{XD} = 3\mathbf{c} - \dfrac{1}{2}\mathbf{a}$

find the value of *k*.

k = ...

(Total for Question 2 is 4 marks)

3

ABCD is a rectangle.

A, *E* and *B* are points on the straight line **L** with equation $x + 2y = 12$
A and *D* are points on the straight line **M**.

$AE = EB$

Find an equation for **M**.

....

(Total for Question 3 is 4 marks)

4 Solve algebraically the simultaneous equations

$$x^2 + y^2 = 25$$
$$y - 3x = 13$$

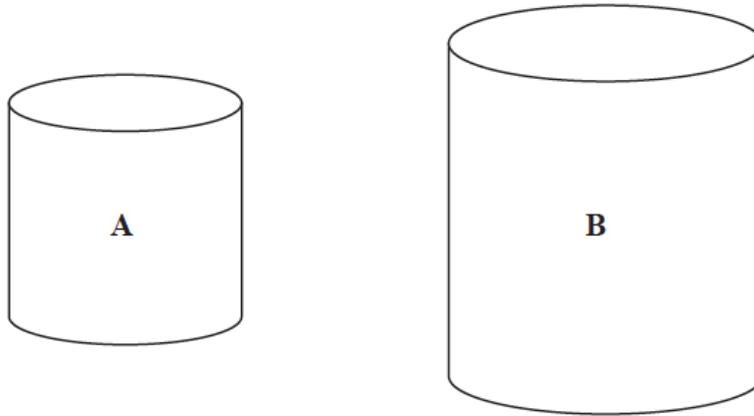..

(Total for Question 4 is 5 marks)

_____

_____

5 Given that

$$x^2 : (3x + 5) = 1 : 2$$

find the possible values of x.

....

(Total for Question 5 is 4 marks)
_____
_____

6 **A** and **B** are two similar cylindrical containers.

the surface area of container **A** : the surface area of container **B** = 4 : 9

Tyler fills container **A** with water.

She then pours all the water into container **B**.
Tyler repeats this and stops when container **B** is full of water.

Work out the number of times that Tyler fills container **A** with water.
You must show all your working.

....

(Total for Question 6 is 4 marks)

7 White shapes and black shapes are used in a game.

Some of the shapes are circles.

All the other shapes are squares.

The ratio of the number of white shapes to the number of black shapes is   3:7

The ratio of the number of white circles to the number of white squares is   4:5

The ratio of the number of black circles to the number of black squares is   2:5

Work out what fraction of all the shapes are circles.

....

(Total for Question 7 is 4 marks)

_____

8 Three solid shapes **A**, **B** and **C** are similar.

The surface area of shape **A** is 4 cm^2
The surface area of shape **B** is 25 cm^2

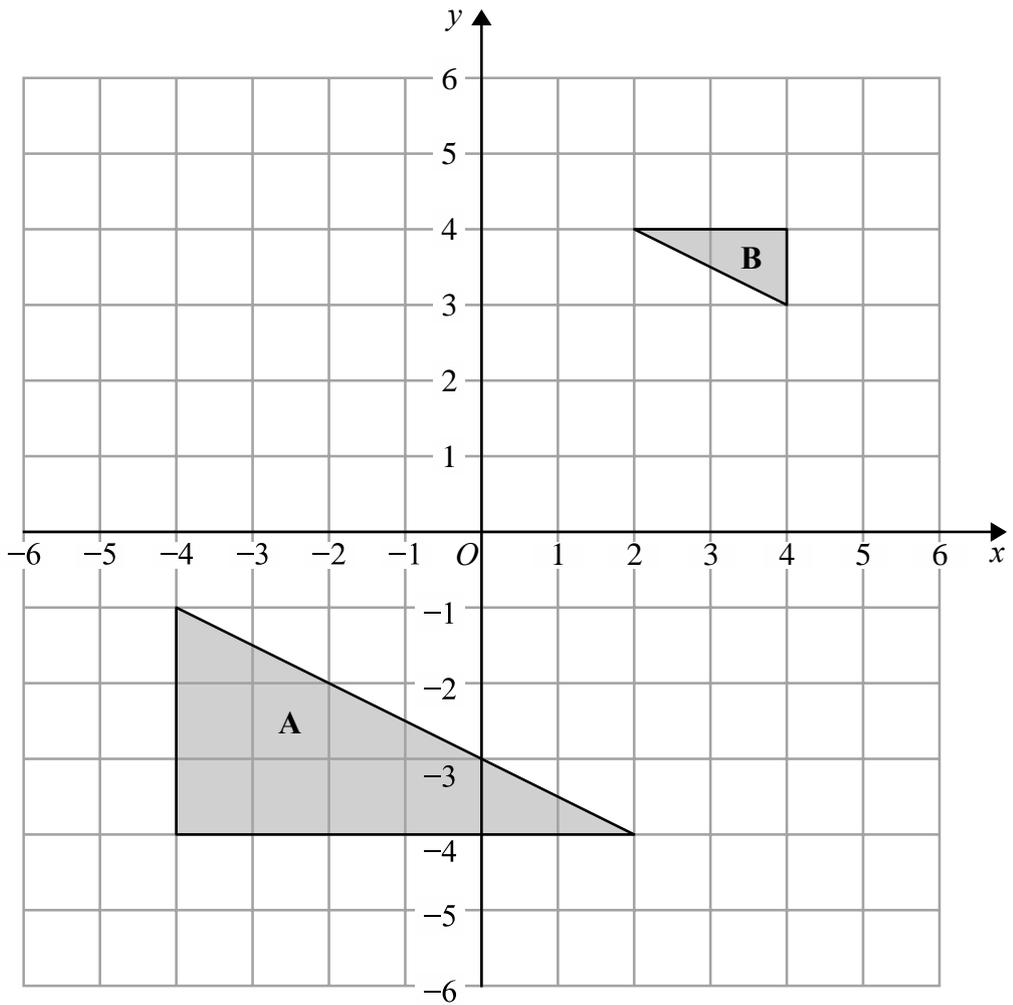The ratio of the volume of shape **B** to the volume of shape **C** is 27 : 64

Work out the ratio of the height of shape **A** to the height of shape **C**.
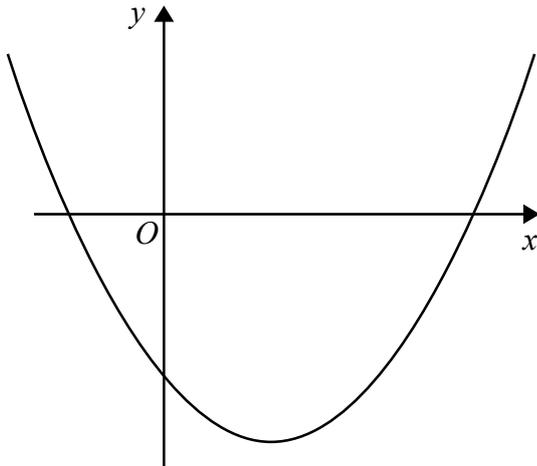Give your answer in its simplest form.

....

(Total for Question 8 is 4 marks)

_____

9

Describe fully the single transformation that maps triangle **A** onto triangle **B**.

....

....

(Total for Question 9 is 2 marks)

_____

_____

10 Here is a sketch of a curve.
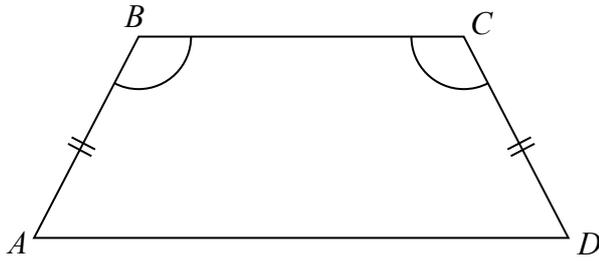

The equation of the curve is $y = x^2 + ax + b$ where a and b are integers.

The points $(0, -5)$ and $(5, 0)$ lie on the curve.

Find the coordinates of the turning point of the curve.

(......................... , ...)

(Total for Question 10 is 4 marks)

_____

11 *ABCD* is a quadrilateral.

AB = CD.
Angle *ABC* = angle *BCD*.

Prove that *AC = BD*.

(Total for Question 11 is 4 marks)

12 Sketch the graph of

$$y = 2x^2 - 8x - 5$$

showing the coordinates of the turning point and the exact coordinates of any intercepts
with the coordinate axes.

(Total for Question 12 is 5 marks)

_____
_____

13   Given that *n* can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

(Total for Question 13 is 2 marks)

_____

14 The table shows some values of *x* and *y* that satisfy the equation $y = a \cos x° + b$

			3			1
			1			1

Find the value of *y* when $x = 45$

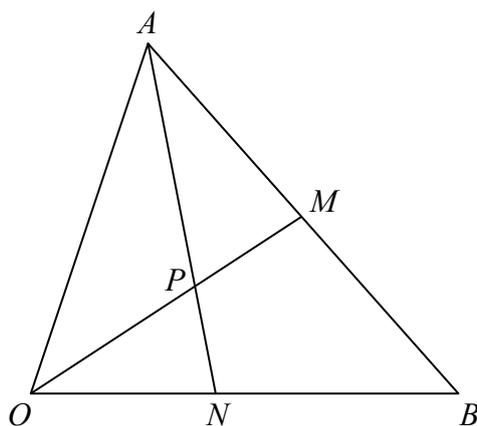...

(Total for Question 14 is 4 marks)

15 The ratio $(y + x) : (y - x)$ is equivalent to $k : 1$

Show that $y = \dfrac{x(k + 1)}{k - 1}$

(Total for Question 15 is 3 marks)

_____

_____

16

OAB is a triangle.
OPM and *APN* are straight lines.
M is the midpoint of *AB*.

$$\overrightarrow{OA} = \mathbf{a} \quad \overrightarrow{OB} = \mathbf{b}$$

OP : *PM* = 3 : 2

Work out the ratio *ON* : *NB*

....

(Total for Question 16 is 5 marks)

_____

17 *A*, *B*, *C* and *D* are four points on a circle.
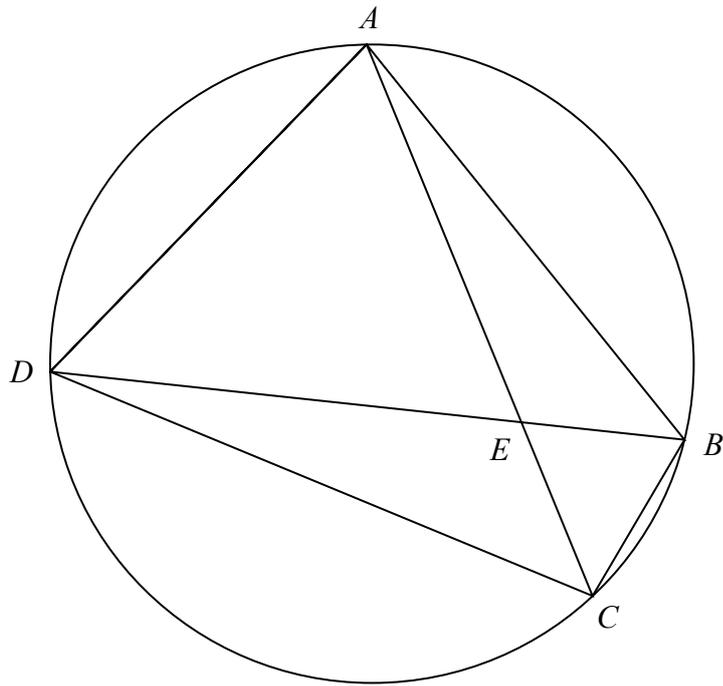
AEC and *DEB* are straight lines.

Triangle *AED* is an equilateral triangle.

Prove that triangle *ABC* is congruent to triangle *DCB*.

(Total for Question 17 is 4 marks)

_____

_____

18   There are only r red counters and g green counters in a bag.

A counter is taken at random from the bag.

The probability that the counter is green is $\dfrac{3}{7}$

The counter is put back in the bag.

2 more red counters and 3 more green counters are put in the bag.
A counter is taken at random from the bag.

The probability that the counter is green is $\dfrac{6}{13}$
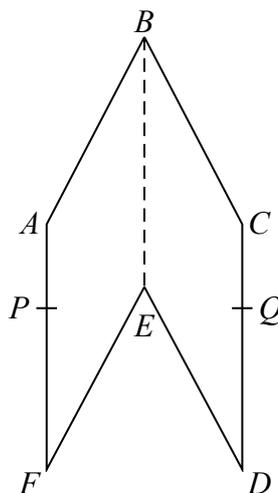
Find the number of red counters and the number of green counters that were in the bag originally.

red counters..

green counters..

(Total for Question 18 is 5 marks)

19 The diagram shows a hexagon *ABCDEF*.

ABEF and *CBED* are congruent parallelograms where $AB = BC = x$ cm.
P is the point on *AF* and *Q* is the point on *CD* such that $BP = BQ = 10$ cm.

Given that angle $ABC = 30°$,

prove that $\qquad \cos PBQ = 1 - \dfrac{(2 - \sqrt{3})}{200} \, x^2$

(Total for Question 19 is 5 marks)

20 There are only green pens and blue pens in a box.

There are three more blue pens than green pens in the box.
There are more than 12 pens in the box.

Simon is going to take at random two pens from the box.

The probability that Simon will take two pens of the same colour is $\dfrac{27}{55}$

Work out the number of green pens in the box.

....

(Total for Question 20 is 6 marks)

TOTAL FOR PAPER IS 80 MARKS

Candidate surname		Other names

Pearson Edexcel
Level 1/Level 2 GCSE (9–1)

Centre Number	Candidate Number

Aiming for 9 – Spring 2020 practice

Morning (Time: 1 hour 30 minutes)	Paper Reference **1MA1/2H**

Mathematics
Paper 2 (Calculator)
Higher Tier

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

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Information

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Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

1 The equation of a curve is $y = a^x$
   A is the point where the curve intersects the y-axis.

   (*a*) State the coordinates of A.

   (.......................... , ...)
   (1)

   The equation of circle **C** is $x^2 + y^2 = 16$

   The circle **C** is translated by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ to give circle **B**.
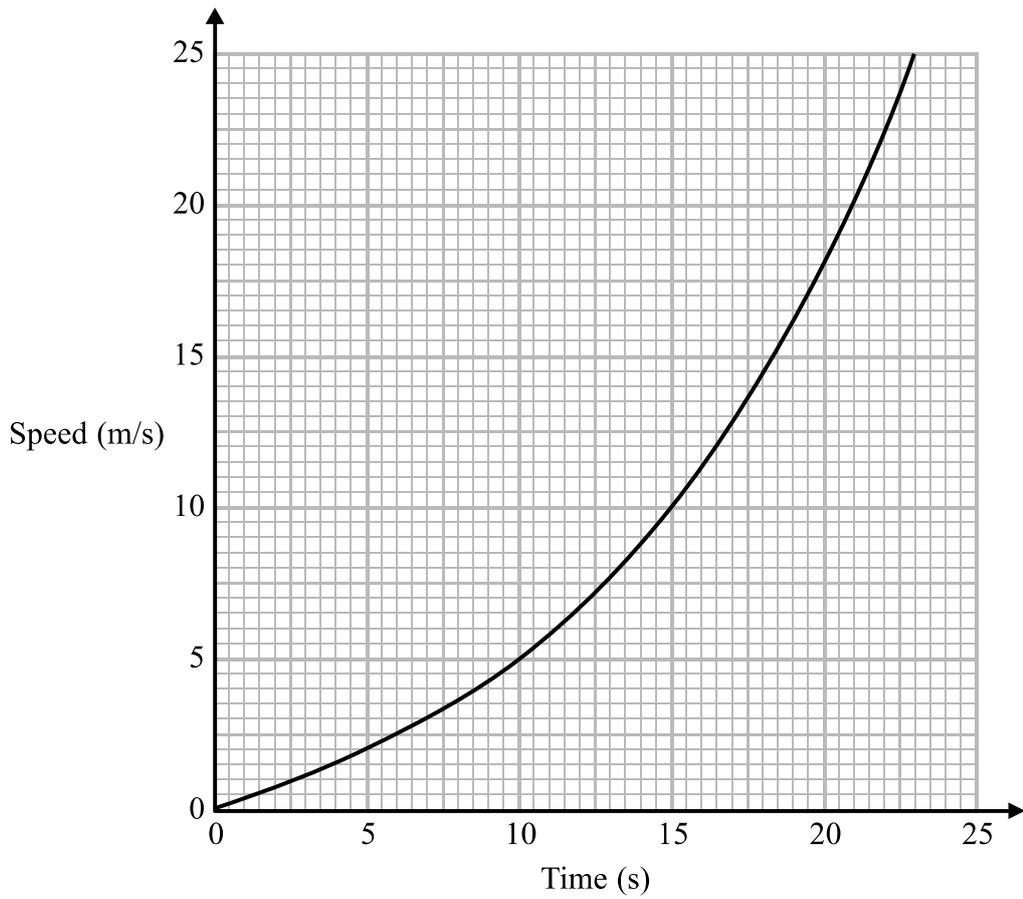
   (*b*) Draw a sketch of circle **B**.

   Label with coordinates

           the centre of circle **B**
           and any points of intersection with the *x*-axis.

   (3)

   (Total for Question 1 is 4 marks)

2 Here is a speed-time graph for a train.

(*a*) Work out an estimate for the distance the train travelled in the first 20 seconds.
Use 4 strips of equal width.

.... m

(3)

(*b*) Is your answer to (*a*) an underestimate or an overestimate of the actual distance the
train travelled?
Give a reason for your answer.

....

....

....

..................**(1)**

(Total for Question 2 is 4 marks)

3 At time $t = 0$ hours a tank is full of water.

Water leaks from the tank.
At the end of every hour there is 2% less water in the tank than at the start of the
hour.

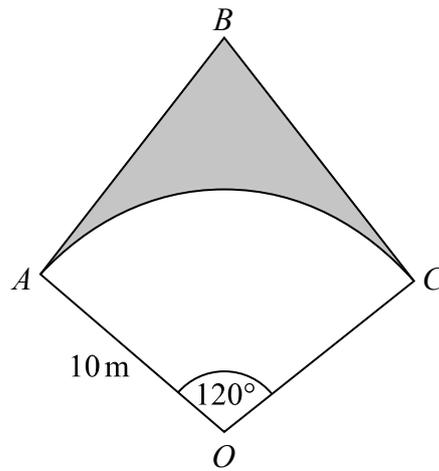The volume of water, in litres, in the tank at time t hours is V_t

Given that

$V_0 = 2000$
$V_{t+1} = kV_t$

write down the value of k.

$k = $ ....

(Total for Question 3 is 1 mark)

_____

4

OAC is a sector of a circle, centre *O*, radius 10 m.

BA is the tangent to the circle at point *A*.
BC is the tangent to the circle at point *C*.

Angle *AOC* = 120°

Calculate the area of the shaded region.
Give your answer correct to 3 significant figures.

....m^2

(Total for Question 4 is 5 marks)

5 Sketch the graph of $y = \tan x°$ for $0 \leq x \leq 360$
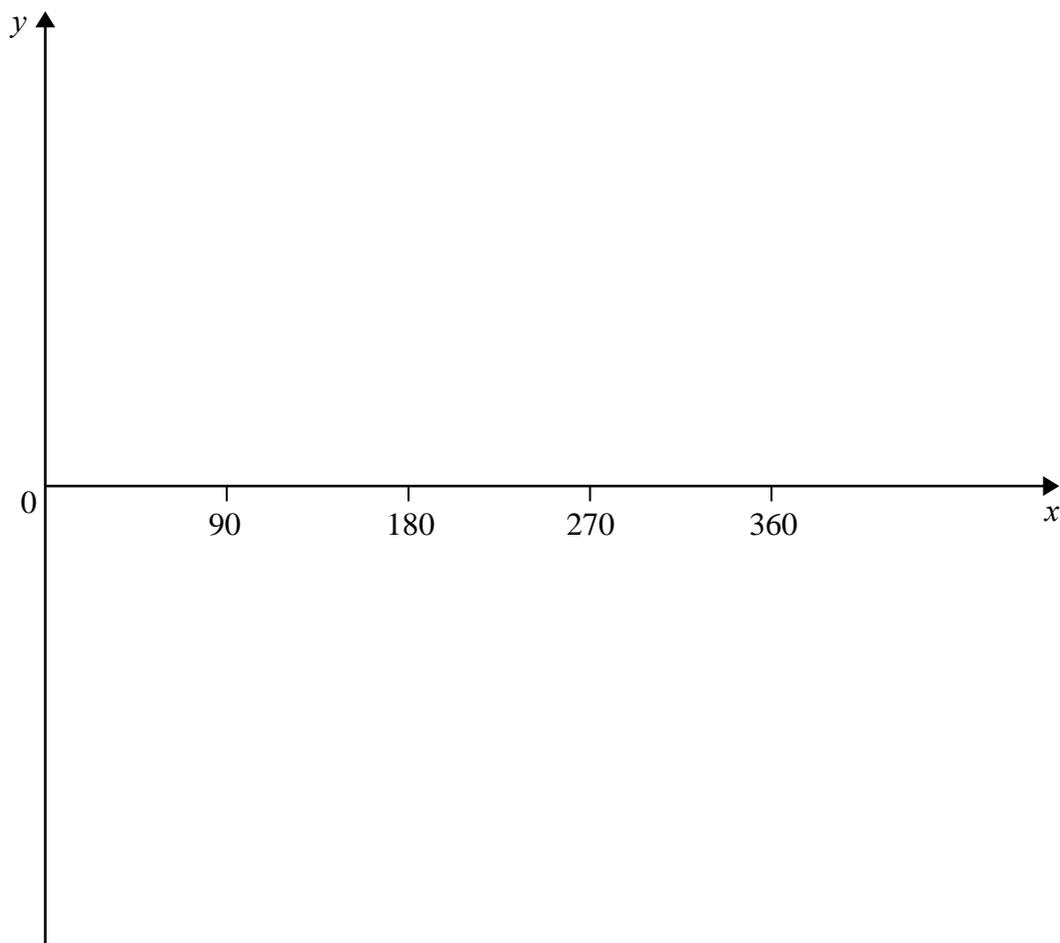
(Total for Question 5 is 2 marks)

6 $2 - \dfrac{x+2}{x-3} - \dfrac{x-6}{x+3}$ can be written as a single fraction in the form $\dfrac{ax+b}{x^2-9}$

where a and b are integers.

Work out the value of a and the value of b.

$a = $ ....

$b = $ ....

(Total for Question 6 is 4 marks)

_____

7 $\qquad 16^{\frac{1}{5}} \times 2^x = 8^{\frac{3}{4}}$

Work out the exact value of *x*.

....

(Total for Question 7 is 3 marks)

_____

_____

8 The distance-time graph shows information about part of a car journey.
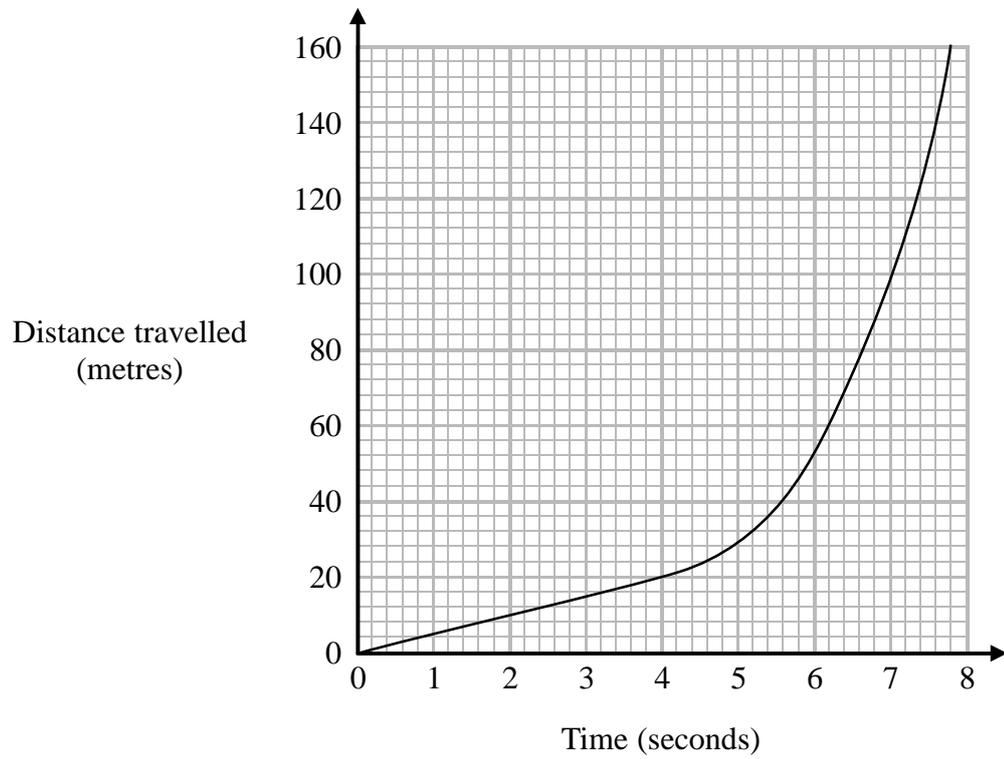
Use the graph to estimate the speed of the car at time 5 seconds.

.... m/s

(**Total for Question 8 is 3 marks**)

9 Here are two similar solid shapes.

A **B**

surface area of shape **A** : surface area of shape **B** = 3 : 4

The volume of shape **B** is 10 cm^3

Work out the volume of shape **A**.
Give your answer correct to 3 significant figures.

.... cm^3

(Total for Question 9 is 3 marks)

10 When a biased coin is thrown 4 times, the probability of getting 4 heads is $\dfrac{16}{81}$

Work out the probability of getting 4 tails when the coin is thrown 4 times.

....

(Total for Question 10 is 2 marks)

_____

11   *A*, *B*, *C* and *D* are four points on the circumference of a circle.
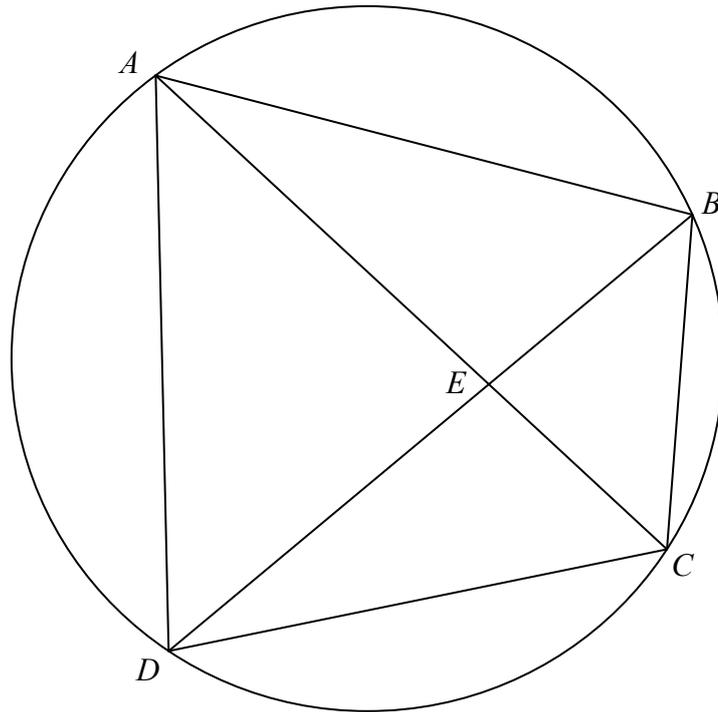
AEC and *BED* are straight lines.

Prove that triangle *ABE* and triangle *DCE* are similar.
You must give reasons for each stage of your working.

(Total for Question 11 is 3 marks)

_____

_____

12 The nth term of a sequence is $\qquad 2n^2 - 1$

The nth term of a different sequence is $\;\; 40 - n^2$

Show that there is only one number that is in both of these sequences.

(Total for Question 12 is 3 marks)

_____

13
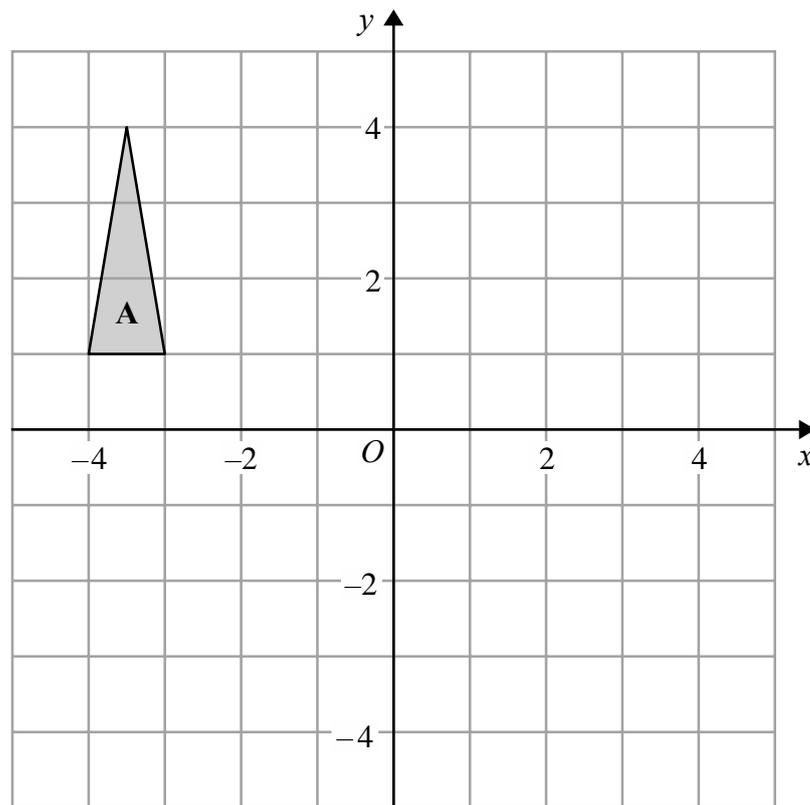
Triangle **A** is transformed by the combined transformation of a rotation of 180° about the

point (−2, 0) followed by a translation with vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

One point on triangle **A** is invariant under the combined transformation.

Find the coordinates of this point.

(............................ , ...)

(Total for Question 13 is 2 marks)

14 Here are two right-angled triangles.
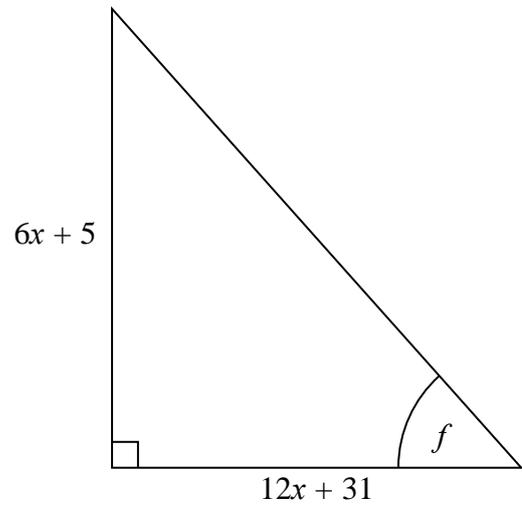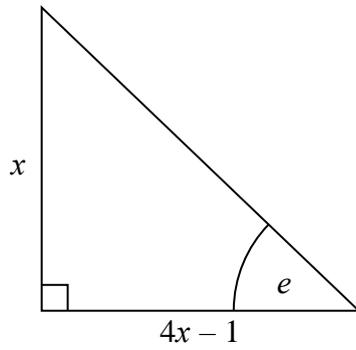
Given that

$$\tan e = \tan f$$

find the value of x.

You must show all your working.

....

(**Total for Question 14 is 5 marks**)

15   The diagram shows 3 identical circles inside a rectangle.
Each circle touches the other two circles and the sides of the rectangle, as shown in
the diagram.
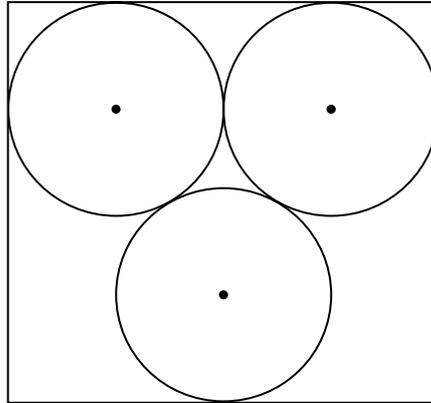
The radius of each circle is 24 mm.

Work out the area of the rectangle.
Give your answer correct to 3 significant figures.

.... mm^2

(Total for Question 15 is 4 marks)

16 The diagram shows the positions of three towns, Acton (*A*), Barston (*B*) and Chorlton (*C*).
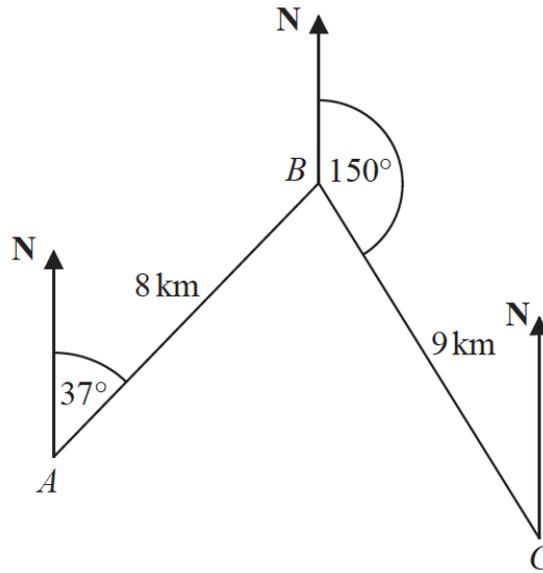
Barston is 8 km from Acton on a bearing of 037°
Chorlton is 9 km from Barston on a bearing of 150°

Find the bearing of Chorlton from Acton.
Give your answer correct to 1 decimal place.
You must show all your working.

....°

(Total for Question 16 is 5 marks)

17 The diagram shows a parallelogram.
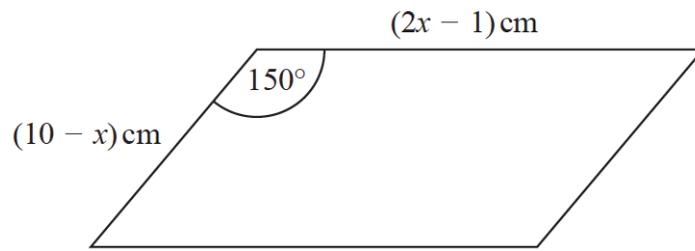
The area of the parallelogram is greater than 15 cm^2

(*a*) Show that $\quad 2x^2 - 21x + 40 < 0$

(3)

(*b*) Find the range of possible values of *x*.

....

(3)

(Total for Question 17 is 6 marks)

18 The diagram shows a rectangle, *ABDE*, and two congruent triangles, *AFE* and *BCD*.
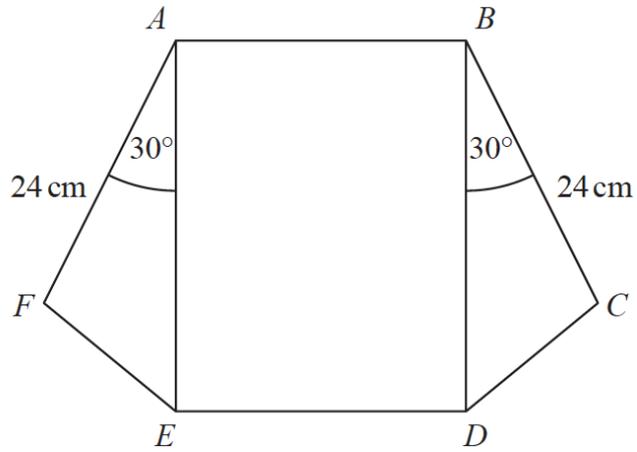
area of rectangle *ABDE* = area of triangle *AFE* + area of triangle *BCD*

$$AB : AE = 1 : 3$$

Work out the length of *AE*.

.... cm

(Total for Question 18 is 4 marks)

19 The diagram shows a circle, centre O.
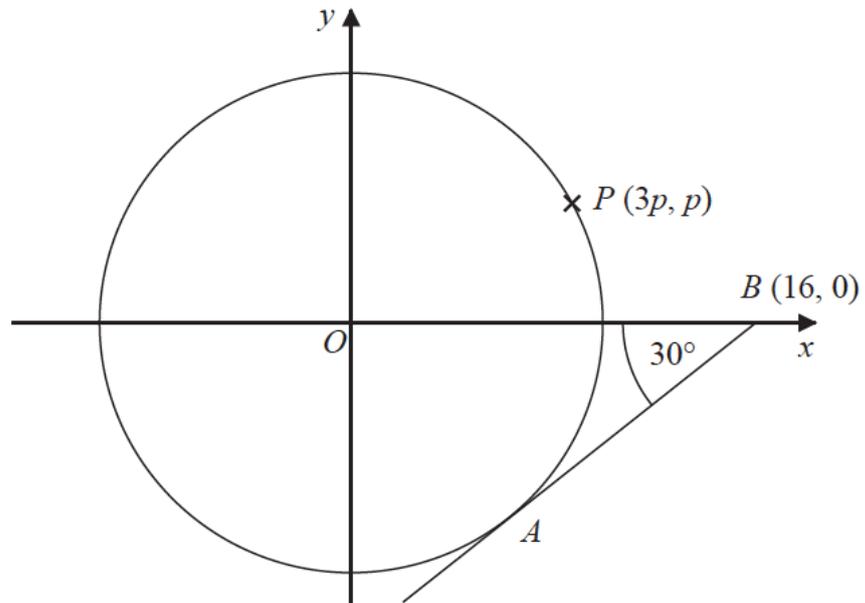
AB is the tangent to the circle at the point *A*.
Angle *OBA* = 30°

Point *B* has coordinates (16, 0)
Point *P* has coordinates (3*p*, *p*)

Find the value of *p*.
Give your answer correct to 1 decimal place.
You must show all your working.

p = ...

(Total for Question 19 is 4 marks)

20
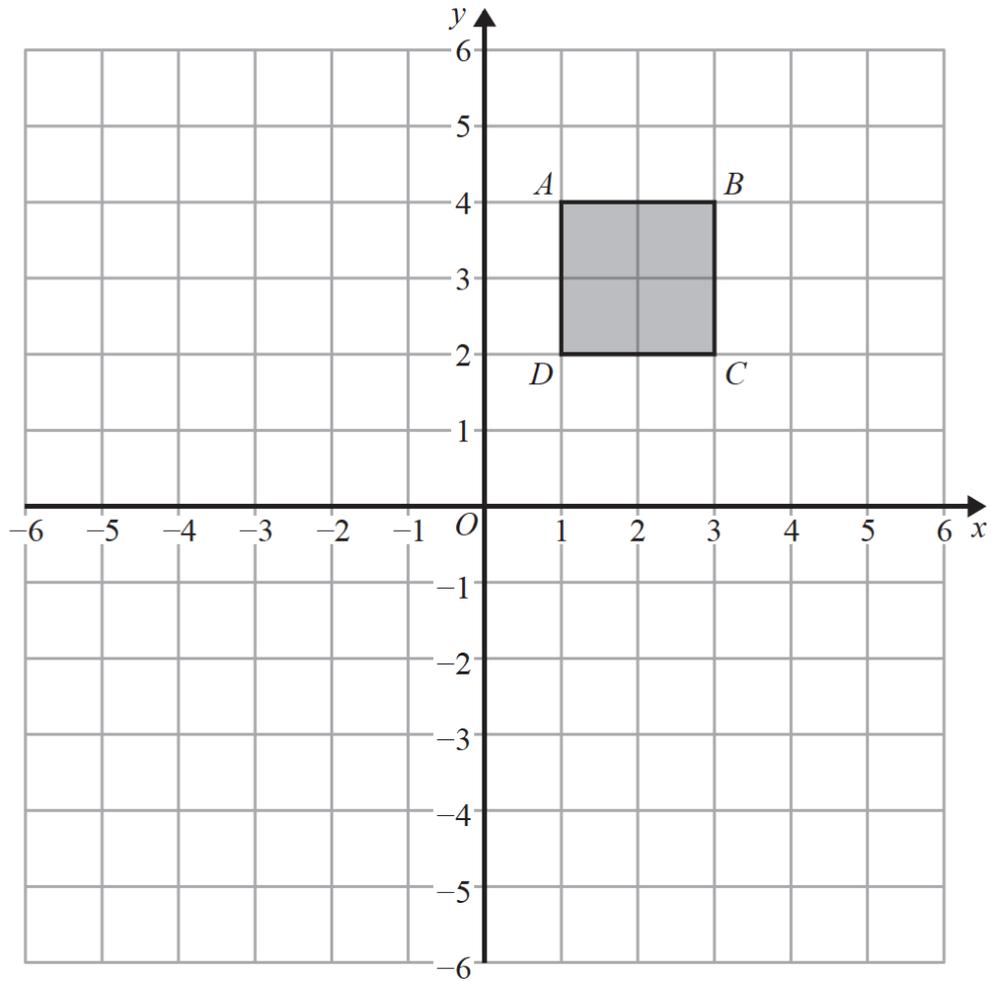
Square *ABCD* is transformed by a combined transformation of a reflection in the
line $x = -1$
followed by a rotation.

Under the combined transformation, two vertices of the square *ABCD* are
invariant.
Describe fully one possible rotation.

....

....

(Total for Question 20 is 2 marks)

21 **L** is the circle with equation $x^2 + y^2 = 4$

$P\left(\dfrac{3}{2}, \dfrac{\sqrt{7}}{2}\right)$ is a point on **L**.

Find an equation of the tangent to **L** at the point P.

....

(Total for Question 21 is 3 marks)

_____

22
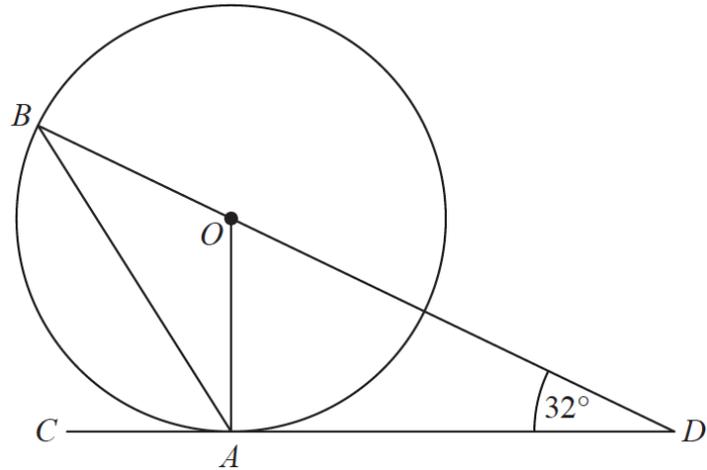
A and *B* are points on a circle with centre *O*.
CAD is the tangent to the circle at *A*.

BOD is a straight line.
Angle *ODA* = 32°

Work out the size of angle *CAB*.
You must show all your working.

....°

(Total for Question 22 is 3 marks)

_____
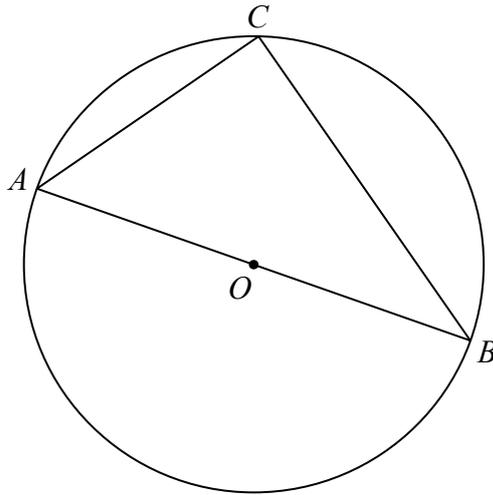
A, *B* and *C* are points on the circumference of a circle, centre *O*.
AOB is a diameter of the circle.

Prove that angle *ACB* is 90°
You must **not** use any circle theorems in your proof.

(Total for Question 23 is 4 marks)

TOTAL FOR PAPER IS 80 MARKS

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Pearson Edexcel
Level 1/Level 2 GCSE (9–1)

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1 The straight line **L** has equation $3x + 2y = 17$

The point A has coordinates $(0, 2)$
The straight line **M** is perpendicular to **L** and passes through A.

Line **L** crosses the y-axis at the point B.
Lines **L** and **M** intersect at the point C.

Work out the area of triangle ABC.
You must show all your working.

....

(Total for Question 1 is 5 marks)

2 p and q are two numbers such that $p > q$

When you subtract 5 from p and subtract 5 from q the answers are in the ratio 5 : 1
When you add 20 to p and add 20 to q the answers are in the ratio 5 : 2

Find the ratio $p : q$
Give your answer in its simplest form.

....

(Total for Question 2 is 5 marks)

3   The pie chart gives information about the ages, in years, of people living in two towns, Adley and Bridford.

Adley

80+

0 – 19

60 – 79

20 – 39

40 – 59
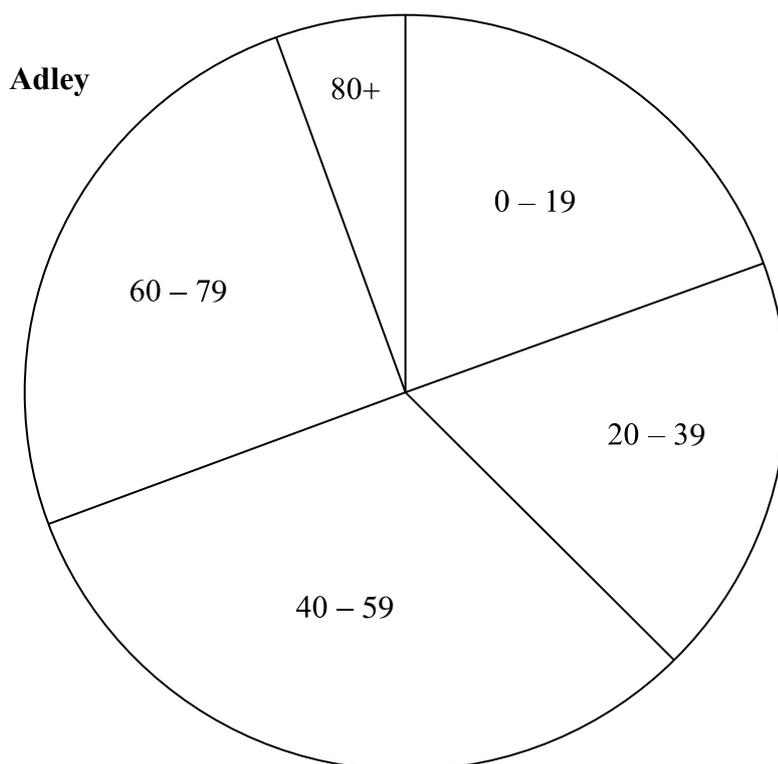
Diagrams accurately drawn

Bridford

80+

0 – 19

60 – 79

20 – 39

40 – 59
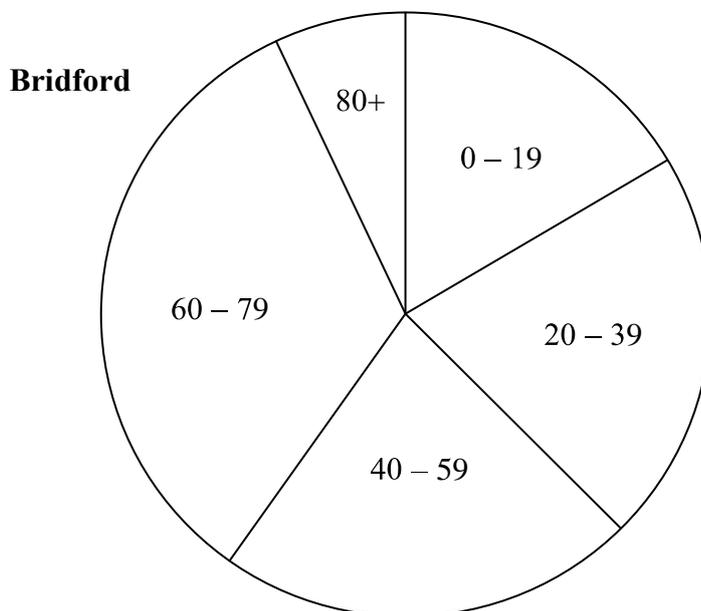
The ratio of the number of people living in Adley to the number of people living in Bridford is given by the ratio of the areas of the pie charts.

What proportion of the total number of people living in these two towns live in Adley **and** are aged 0 – 19?
Give your answer correct to 3 significant figures.

....

(Total for Question 3 is 3 marks)

4   The points *A*, *B*, *C* and *D* lie in order on a straight line.

$$AB : BD = 1{:}5$$

$$AC : CD = 7{:}11$$

Work out *AB* : *BC* : *CD*

.... : .... : ....

(Total for Question 4 is 3 marks)

_____

5   *ABC* and *ADC* are triangles.
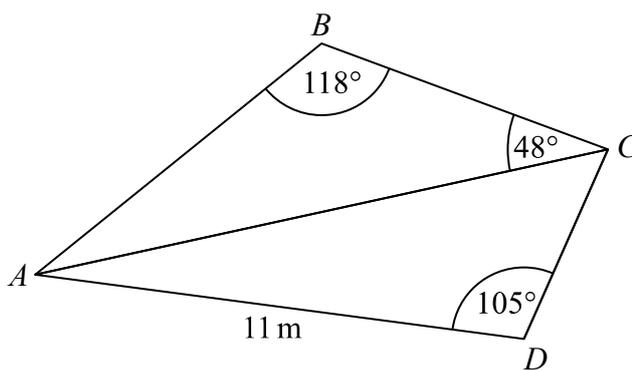
The area of triangle *ADC* is 56 m²

Work out the length of *AB*.
Give your answer correct to 1 decimal place.

.... m

(Total for Question 5 is 5 marks)

6   The function f and g are such that

$$f(x) = 5x + 3 \qquad g(x) = ax + b \quad \text{where } a \text{ and } b \text{ are constants.}$$

$g(3) = 20$ and $f^{-1}(33) = g(1)$

Find the value of a and the value of b.

$a = $ ....

$b = $ ....

(Total for Question 6 is 5 marks)

7 The diagram shows triangle *ABC*.
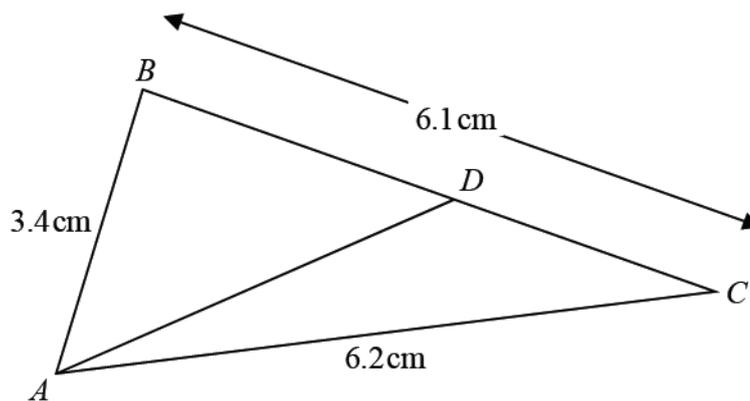
AB = 3.4 cm $\qquad$ *AC* = 6.2 cm $\qquad$ *BC* = 6.1 cm

D is the point on *BC* such that

$$\text{size of angle } DAC = \frac{2}{5} \times \text{size of angle } BCA$$

Calculate the length *DC*.

Give your answer correct to 3 significant figures.
You must show all your working.

....cm

(Total for Question 7 is 5 marks)

8   50 people were asked if they speak French or German or Spanish.

Of these people,
   31 speak French
   2 speak French, German and Spanish
   4 speak French and Spanish but not German
   7 speak German and Spanish
   8 do not speak any of the languages
   all 10 people who speak German speak at least one other language

Two of the 50 people are chosen at random.

Work out the probability that they both only speak Spanish.

....

(**Total for Question 8 is 5 marks**)

9   Cone **A** and cone **B** are mathematically similar.
The ratio of the volume of cone **A** to the volume of cone **B** is 27 : 8

The surface area of cone **A** is 297 cm^2

Show that the surface area of cone **B** is 132 cm^2

(Total for Question 9 is 3 marks)
_____

10   Prove algebraically that the straight line with equation $x - 2y = 10$ is a tangent to the circle with equation $x^2 + y^2 = 20$

(Total for Question 10 is 5 marks)

11   Marek has 9 cards.
There is a number on each card.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Marek takes at random two of the cards.
He works out the product of the numbers on the two cards.

Work out the probability that the product is an even number.

....

(Total for Question 11 is 3 marks)

12   Sally used her calculator to work out the value of a number *y*.

The answer on her calculator display began

8.3

Complete the error interval for *y*.

.... $\leq y <$ ....

(Total for Question 12 is 2 marks)

13 The diagram shows part of the graph of $\quad y = x^2 - 2x + 3$
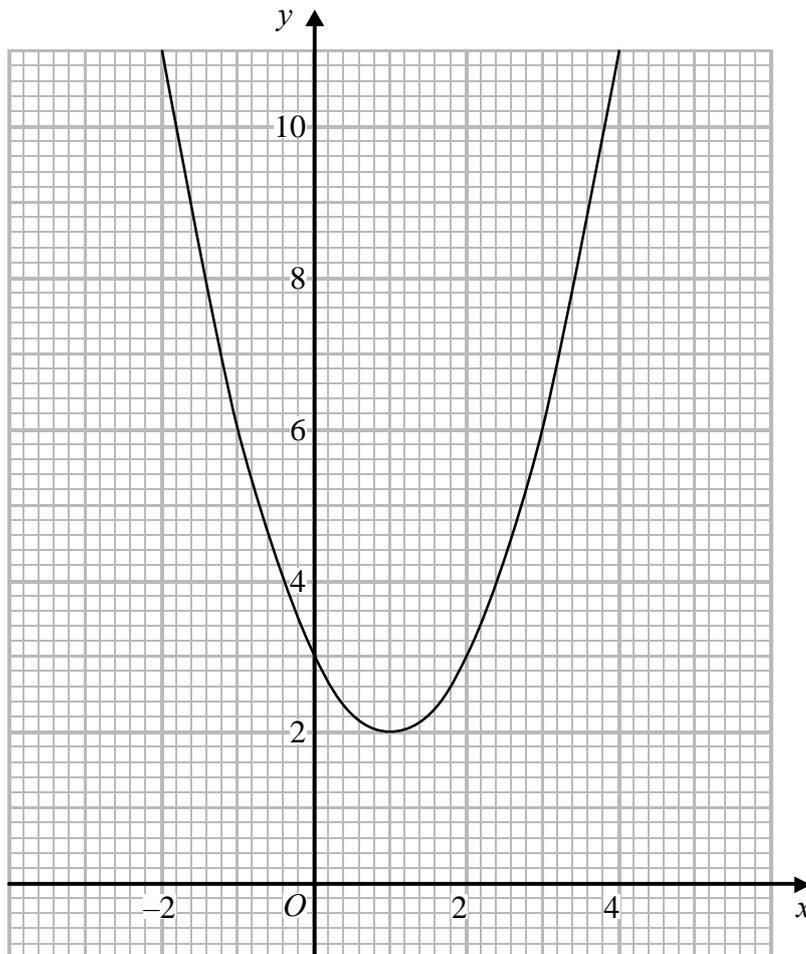
(*a*) By drawing a suitable straight line, use your graph to find estimates for the solutions of $x^2 - 3x - 1 = 0$

....

(2)

P is the point on the graph of $y = x^2 - 2x + 3$ where $x = 2$

(*b*) Calculate an estimate for the gradient of the graph at the point *P*.

....

(3)

(Total for Question 13 is 5 marks)

14   Solve algebraically the simultaneous equations

$$x^2 - 4y^2 = 9$$
$$3x + 4y = 7$$

....

(Total for Question 14 is 5 marks)

$$x^2 - 4y^2 = 9$$

$$3x + 4y = 7$$

15

N

A

P

O   M   B
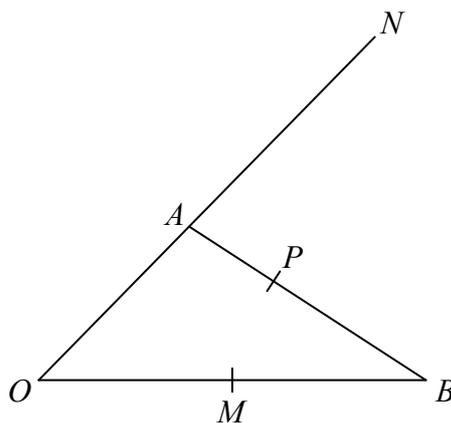
OAN, *OMB* and *APB* are straight lines.
AN = 2*OA*.
M is the midpoint of *OB*.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

$\overrightarrow{AP} = k\ \overrightarrow{AB}$ where *k* is a scalar quantity.

Given that *MPN* is a straight line, find the value of *k*.

....

(Total for Question 15 is 5 marks)
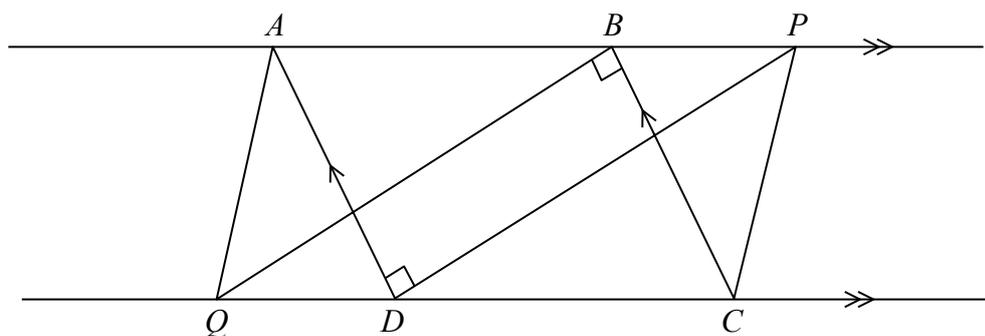
ABCD is a parallelogram.
ABP and *QDC* are straight lines.
Angle *ADP* = angle *CBQ* = 90°

(*a*) Prove that triangle *ADP* is congruent to triangle *CBQ*.

(**3**)

(*b*) Explain why *AQ* is parallel to *PC*.

(**2**)

(**Total for Question 16 is 5 marks**)

_____

17   S is a geometric sequence.

(*a*) Given that ($\sqrt{x}$ – 1), 1 and ($\sqrt{x}$ + 1) are the first three terms of S, find the value of *x*.
   You must show all your working.

....
(3)

(*b*) Show that the 5th term of S is $7 + 5\sqrt{2}$

(2)

(Total for Question 17 is 5 marks)

18 Martin truncates the number N to 1 digit.
The result is 7

Write down the error interval for N.

....

(Total for Question 18 is 2 marks)

19
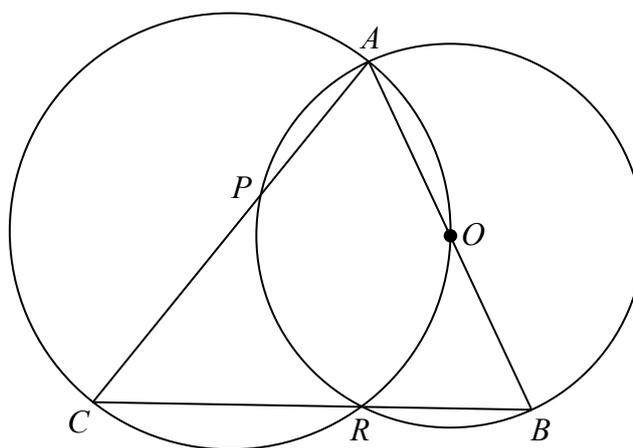
A, *B*, *R* and *P* are four points on a circle with centre *O*.
A, *O*, *R* and *C* are four points on a different circle.
The two circles intersect at the points *A* and *R*.

CPA, *CRB* and *AOB* are straight lines.

Prove that angle *CAB* = angle *ABC*.

(Total for Question 19 is 4 marks)

TOTAL FOR PAPER IS 80 MARKS

Aiming for Grade 9 – Spring 2020 student-friendly mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn't show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.

NOTES ON MARKING PRINCIPLES

Guidance on the use of codes within this mark scheme
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.
P1 – process mark. This mark is generally given for setting up an appropriate process to find a solution in the context of the question.
A1 – accuracy mark. This mark is generally given for a correct answer following correct working.
B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.

C1 – communication mark. This mark is given for explaining your answer or giving a conclusion in context supported by your working.

Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question 1 (Total 2 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\frac{1}{2}(n^2 + n + n^2 + n + 2n + 2)$	1	This mark is given for expanding the brackets
	$= \frac{1}{2}(2n^2 + 4n + 2)$ $= n^2 + 2n + 1$ $= (n + 1)^2$ which is square for all integer values of n	1	This mark is given for a complete proof with reference to $(n + 1)^2$ being square for all integers n

Question 2 (Total 3 marks)

Part	Working an or answer examiner might expect to see	Mark	Notes
	$\overline{OX} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$	P1	This mark is given for the first step to solve the problem
	$\overline{OD} = \overline{OX} + \overline{XD}$ $= \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c} + 3\mathbf{c} - \frac{1}{2}\mathbf{a}$ $\overline{CD} = \overline{OD} - \overline{OC}$ $= 3.5\mathbf{c} - \mathbf{c} = 2.5\mathbf{c}$	P1	This mark is given for a process to find a vector expression for $\overline{CD}$
	$\overline{OC} : \overline{CD} = k : 1 = \mathbf{c} + 2.5\mathbf{c}$	P1	This mark is given for a process to find the value of k (using ratios)

		$k = \dfrac{1}{2.5} = \dfrac{2}{5}$	A1	This mark is given for the correct answer only

Question 3 (Total 4 marks)

		Working or answer an examiner might expect to see		Notes
		$B = (12, 0)$, $E = (0, 6)$ Gradient of **L** $= -\dfrac{1}{2}$		This mark is given for rearranging to find a gradient or positions of B and E
		$A = (-12, 12)$		This mark is given for finding the position of A
		Gradient of **M** $= 2$		This mark is given for a finding the gradient of **M**
		Equation of **M** is $y = 2x + 36$		This mark is given for the correct answer only

Question 4 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$y - 3x = 13$, so $y = 3x + 13$ $x^2 + (3x + 13)^2 = 25$	M1	This mark is given for the substitution of $y = 3x + 13$ into $x^2 + y^2 = 25$
	$x^2 + 9x^2 + 39x + 39x + 169 = 25$	M1	This mark is given for the expansion of $x^2 + (3x + 13)^2 = 25$
	$10x^2 + 78x + 144 = 0$	M1	This mark is given for forming a quadratic equation equal to zero

	Working or answer an examiner might expect to see	Mark	Notes
	$2(5x^2 + 39x + 72)$ $= 0$ $2(5x + 24)(x + 3) =$ 0	M1	This mark is given for a method to solve the quadratic equation
	$x = -3, \quad y = 4$ $x = -\dfrac{24}{5}, \; y = -\dfrac{7}{5}$	A1	This mark is given for a pair of correct solutions only

Question 5 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\dfrac{x^2}{3x + 5} = \dfrac{1}{2}$	P1	This mark is given for a process to form an equation
	$2x^2 = 3x + 5$ $2x^2 - 3x - 5 = 0$	P1	This mark is given for a process to write a quadratic equation to be solved
	$(2x - 5)(x + 1) = 0$	P1	This mark is given for a process to factorise the quadratic equation
	$x = 2.5,\ x = -1$	A1	This mark is given for the correct answers only

Question 6 (Total 4 marks)

		Working or answer an examiner might expect to see		Notes
		Ratio of lengths of containers = $\sqrt{4} : \sqrt{9} = 2 : 3$		This mark is given for a process to find corresponding lengths of the containers
		Ratio of volumes of containers = $2^3 : 3^3 = 8 : 27$		This mark is given for a process to find corresponding volumes of the containers
		$\dfrac{27}{8}$		This mark is given for a process to find how much bigger container **B** is than container **A**
		$\dfrac{27}{8}$ = 3.375 so 4 times		This mark is given for the correct number of times Tyler fills container **A**

Question 7 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\dfrac{3}{3+7} \times \dfrac{4}{4+5} =$ $\dfrac{3}{10} \times \dfrac{4}{9} =$ $\dfrac{12}{90} = \dfrac{2}{15}$	P1	This mark is given for a process to find the proportion of shapes which are white circles
	$\dfrac{7}{3+7} \times \dfrac{2}{2+5} =$ $\dfrac{7}{10} \times \dfrac{2}{7} =$ $\dfrac{2}{10} = \dfrac{1}{5}$	P1	This mark is given for a process to find the proportion of shapes which are black circles
	$\dfrac{2}{15} + \dfrac{1}{5}$	P1	This mark is given for a process to find the proportion of

			all shapes which are circles
$\dfrac{1}{3}$		A1	This mark is given for the correct answer only

Question 8 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	√4 : √25 = 2 : 5	P1	This mark is given for a process to find the ratio of the lengths **A**:**B**
	$\sqrt[3]{27}$: $\sqrt[3]{64}$ = 3 : 4	P1	This mark is given for a process to find the ratio of the lengths **B**:**C**
	6 : 15 : 20	P1	This mark is given for a process to find the ratio of the lengths **A**:**B**:**C**
	3 : 10	A1	This mark is given for the correct answer only

Question 9 (Total 2 marks)

P	Working an or answer examiner might expect to see	M	Notes
		C	This mark is given for all two parts of the transformation

		C	This mark is given for all three parts of the transformation described

Enlargement of scale factor $-\dfrac{1}{3}$

with centre (2, 2)

Question 10 (Total 4 marks)
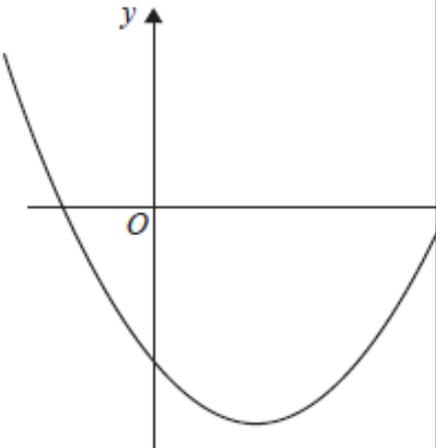
P	Working or answer an examiner might expect to see	M	Notes
	When $x = 0$ and $y = -5$, $-5 = 0^2 + (a \times 0) + b$ $b = -5$	P	This mark is given for a process to substitute to find the value of b
	When $x = 5$ and $y = 0$, $0 = 5^2 + 5a - 5$ $a = -4$	P	This mark is given for a process to

				substitute to find the value of a
	$y = x^2 - 4x - 5 = (x + 1)(x - 5)$ Thus the other intercept is at (–1, 0)		P	This mark is given for a complete process to find the turning point
	Midpoint x-coordinate is 2 When $x = 2$, y-coordinate is –9 Turning point is (2, –9)		A	This mark is given for the correct answer only

Question 11 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$ABC = BCD$	C1	This mark is given for comparing triangles ABC and BCD (given in the diagram)
	$AB = CD$	C1	This mark is given for stating that line $AB = CD$ (given in the diagram)
	$BC = BC$ (common)	C1	This mark is given for stating that line BC is common to both triangles ABC and BCD
	$ABC \equiv DCB$ [SAS] and thus $AC = BD$	C1	This mark is given for a conclusion to the proof showing that the two triangles have a common angle and two common sides; thus $AC = BD$

Question 12 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$x = \dfrac{--8 \pm \sqrt{(-8)^2 - 4 \times 2 \times -5}}{2 \times 2}$	M1	This mark is given for a method to find the

				roots of $y = 0$
	$x = 2 + \sqrt{\dfrac{13}{2}}, \quad 2 - \sqrt{\dfrac{13}{2}}$		M1	This mark is given for finding the roots of $y = 0$
	x-coordinate for turning point = $\dfrac{1}{2}(2 + \sqrt{\dfrac{13}{2}} + 2 - \sqrt{\dfrac{13}{2}}) = 2$ When $x = 2$, $y = -13$		M1	This mark is given for the turning point of $y = 2x^2 - 8x - 5$
			C2	These marks are given for a fully correct parabola drawn with axes labelled, a turning point at (2, –13) and intercepts at (0, –5), $(2 + \sqrt{\dfrac{13}{2}}, 0)$ and $(2 - \sqrt{\dfrac{13}{2}}, 0)$ clearly shown

Question 13 (Total 2 marks)

Part	Working an or answer examiner might expect to see	Mark	Notes
	For even numbers $2n$: $(2n)^2 - 2n = 4n^2 - 2n = 2(2n^2 - n)$ so even For odd numbers $2n + 1$: $(2n + 1)^2 - 2n + 1 = 4n^2 + 4n + 1 - (2n + 1)$ $= 4n^2 + 2n$ $= 2(n^2 - n)$ so even Thus for all integer value of n, $n^2 - n$ is never an odd number	C2	This mark is given for a fully correct proof (C1 is given for a partial explanation)

Question 14 (Total 4 marks)

	Working or answer an examiner might expect to see		Notes
	When $x = 90$, $a \cos x° = 0$ so $b = 1$		This mark is given for finding the value of b
	When $x = 0$, $a \cos x° = 2$ so $a = 2$		This mark is given for finding the value of a
	When $x = 45$, $2 \cos x° + 1 = 2 \times \dfrac{1}{\sqrt{2}} + 1$		This mark is given for finding an expression for $2 \cos x°$ + 1 when $x = 45$
	$1 + \sqrt{2}$		This mark is given for the correct answer only

Question 15 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$(y + x) = k(y − x)$	1	This mark is given for setting up an equations from the information given
	$ky − y = x + kx$	1	This mark is given for isolating x and y on opposite sides
	$y(k − 1) = k(x + 1)$ so $y = \dfrac{k(x+1)}{k-1}$	1	This mark is given for using correct algebra to reach a conclusion

Question 16 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$AB = \mathbf{b} - \mathbf{a}$	P1	This mark is given for the start to a process by finding the vector AB
	$OM = OA + \dfrac{1}{2} AB$ $= \mathbf{a} + \dfrac{1}{2}(\mathbf{b} - \mathbf{a})$	P1	This mark is given for a process to find the vector OM
	$AP = AO + \dfrac{3}{5} OM$ $= -\mathbf{a} + \dfrac{3}{5}(\mathbf{a} + \dfrac{1}{2}(\mathbf{b} - \mathbf{a}))$ $= -\mathbf{a} + \dfrac{3}{5}(\dfrac{1}{2}\mathbf{a} + \dfrac{1}{2}\mathbf{b})$ $= -\dfrac{7}{10}\mathbf{a} + \dfrac{3}{10}\mathbf{b}$	P1	This mark is given for a process to find the vector AP
	$AN = -\mathbf{a} + k\mathbf{b} = \dfrac{10}{7} AP$ $= \dfrac{10}{7}(-\dfrac{7}{10}\mathbf{a} + \dfrac{3}{10}\mathbf{b}) = -\mathbf{a} + \dfrac{3}{7}\mathbf{b}$	P1	This mark is given for a process to find $k = \dfrac{3}{7}$ using AN as a multiple of AP
	Thus $ON : NB = 3 : 4$	A1	This mark is given for the correct answer only

Question 17 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\angle ACB = \angle ADB = 60°$ Angles in the same segment are equal $\angle DBC = \angle DAC = 60°$ Angles in the same segment are equal Thus $\angle ACB = \angle DBC = 60°$	C1	This mark is given for arguments to show that $\angle ACB = \angle ADB$ and $\angle DBC = \angle DAC$ with reasons given to show that $\angle ACB = \angle DBC$
	$\angle ABC = 60 + \angle ABD = 60 + \angle ACD = \angle DCB$ Angles in the same segment are equal	C1	This mark is given for an argument to show that $\angle ABC = \angle DCB$
	BC is common to both triangles	C1	This mark is given for finding a side common to both triangles
	Thus triangles ABC and DCB are congruent (AAS)	C1	This mark is given for a correct conclusion with reference to AAS

Question 18 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes

	$\dfrac{g}{r+g} = \dfrac{3}{7}$	P1	This mark is given for a process to find an initial relationship between r and g
	$\dfrac{g+3}{r+2+g+3} = \dfrac{6}{13}$	P1	This mark is given for a process to find a second relationship between r and g
	$7g = 3(r+g) = 3r + 3g$ $4g = 3r$ $13(g+3) = 6(r+g+5)$ $13g + 39 = 6r + 6g + 30$ $7g + 9 = 6r$	P1	This mark is given for simplifying the expressions for the relationships found
	$4g = 3r$ $7g + 9 = 6r$ $7g + 9 = 8g$	P1	This mark is given for a process to form simultaneous equations to be solved
	$g = 9$ $r = 12$	A1	This mark is given for the correct answer only (12 red, 9 green)

Question 19 (Total 2 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\cos 30° = \dfrac{\sqrt{3}}{2}$	B1	This mark is given for a correct statement about the value of cos 30° (seen anywhere)
	$PQ^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos PBQ$ $= 200 - 200 \cos PBQ$	M1	This mark is given for applying the cosine rule to find an expression for PQ^2
	$AC^2 = x^2 + x^2 - 2 \times x \times x \times \cos 30°$ $= 2x^2 - 2x^2 \dfrac{\sqrt{3}}{2}$ $= 2x^2(1 - \dfrac{\sqrt{3}}{2})$ $= (2 - \sqrt{3})x^2$	M1	This mark is given for applying the cosine rule to find an expression for AC^2
	$\cos PBQ = \dfrac{200 - PQ^2}{200} = 1 - \dfrac{PQ^2}{200}$	M1	This mark is given for rearranging to find an expression for cos PBQ
	$\cos PBQ = 1 - \dfrac{PQ^2}{200} = 1 - \dfrac{AC^2}{200}$ $= 1 - \dfrac{(2 - \sqrt{3})}{200}x^2$	A1	This mark is given for a conclusion of proof with all working seen

Question 20 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	Let x be the number of green pens in the box The probability of taking two green pens is $\dfrac{x}{2x+3} \times \dfrac{x-1}{2x+2}$ The probability of taking two blue pens is $\dfrac{x+3}{2x+3} \times \dfrac{x+2}{2x+2}$	P1	This mark is given for a process to find the probability of taking two green pens or the probability of taking two blue pens
	The probability of taking two pens of the same colour is $\dfrac{x}{2x+3} \times \dfrac{x-1}{2x+2} + \dfrac{x+3}{2x+3} \times \dfrac{x+2}{2x+2} = \dfrac{27}{55}$	P1	This mark is given for forming an equation for the probability Simon takes two pens of the same colour
	$55(x(x-1) + (x+3)(x+2))$ $= 27(2x+3)(2x+2)$	P1	This mark is given for a process to eliminate fractions from the algebraic expression
	$55(2x^2 + 4x + 6) = 27(4x^2 + 10x + 6)$ $110x^2 + 22x + 330 = 108x^2 + 270x + 162$ $2x^2 - 50x + 168 = 0$	P1	This mark is given for reducing the expression to a quadratic equation

	$x^2 - 25x + 84 = 0$		
	$(x - 21)(x - 4) = 0$	P1	This mark is given for finding a method to solve the quadratic equation
	21	A1	This mark is given for the correct answer only

Question 1 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	The curve cuts the y axis at $x = 0$ $y = a^x = a^0 = 1$ (0, 1)	B1	This mark is given for the correct answer only
(b)		M1	This mark is given for any one of a circle with radius 4, centre (3, 0) or points (−1, 0) and (7, 0) labelled
		M1	This mark is given for any further element of a circle with radius 4, centre (3, 0) or points (−1, 0) and (7, 0) labelled
		A1	This mark is given for a fully correct sketch only: a circle with radius 4 and centre (3, 0) and with the points (−1, 0) and (7, 0) labelled
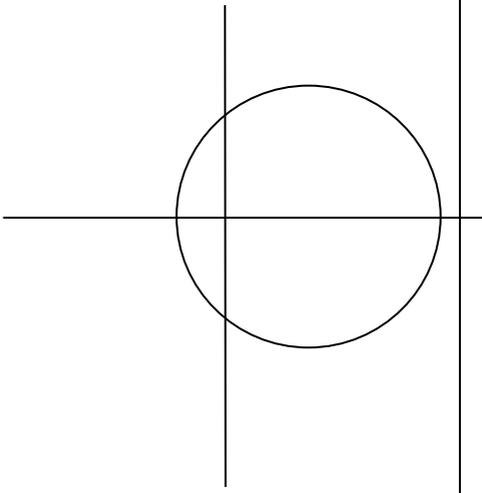
Question 2 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$0.5 \times 5 \times 2 = 5$ or $0.5 \times 5 \times (2 + 5) = 17.5$ or	1	This mark is given for splitting the area into 4 strips and finding the area of

Part	Working or answer an examiner might expect to see	Mark	Notes
	$0.5 \times 5 \times (5 + 10) = 37.5$ or $0.5 \times 5 \times (10 + 18) = 70$		one triangle or trapezium
	$5 + 17.5 + 37.5 + 70$	1	This mark is given for a method to find and add up the totals of the four shapes
	130	1	This mark is given for the correct answer only
(b)	My answer to part (a) is an overestimate because the area measured is greater than the area below the curve	1	This mark is given for a correct statement

Question 3 (Total 1 mark)

	Working or answer an examiner might expect to see		Notes
	$1 - 0.02 = 0.98$		This mark is given for the correct answer only

Question 4 (Total 5 marks)

Part	Working an or answer examiner might expect to see	Mark	Notes
	$OAB = 90°$; $OCB = 90°$	1	This mark is given for identifying a right angle in the diagram
	$AB = CB = 10 \times \tan 60°$ $= 10\sqrt{3}$	1	Thismark is given for finding the length of AB or CB

	Area $OAC = \dfrac{120}{360} \times \pi \times$ $10^2 = 104.72\ldots$	1	This mark is given finding the area of the sector
	Area $OAB = OBC$ $= \dfrac{1}{2} \times 10 \times 10\sqrt{3} =$ $50\sqrt{3}$	1	This mark is given for finding the area of the right angled triangle OAB or OBC
	Shaded area = area $OABC$ – area OAC $= (2 \times 50\sqrt{3}) - 104.72$ $= 68.5$	1	This mark is given for a correct answer in the range 68.4 – 68.6

Question 5 (Total 2 marks)

	Working or answer an examiner might expect to see		**Notes**
			These marks are given for a fully correct sketch between 0° and 360° (C1 is given for a partially correct shape drawn)

Question 6 (Total 4 marks)

	Working an or answer examiner might expect to see		Notes
	$$\frac{2(x-3)(x+3)-(x+2)(x+3)-(x-6)(x-3)}{(x-3)(x+3)}$$		This mark is given for at least two correct terms in rearranging the equation
	$$\frac{2x^2-18-(x^2+5x+6)-(x^2-9x+18)}{(x-3)(x+3)}$$		This mark is given for the correct expansion of at least two expressions
	$$\frac{2x^2-18x-x^2-5x-6-x^2+9x-18}{(x-3)(x+3)}$$		This mark is given for collecting terms
	$$=\frac{4x-42}{(x^2-9)}\ ,\ \text{so}\quad a=4,\ \ b=-42$$		This mark is given for the correct answer only

Question 7 (Total 3 marks)

	Working or answer an examiner might expect to see		Notes
	$16 = 2^4\quad$ or $\quad 8 = 2^3$		This mark is given for converting to a common base with at least one correct conversion
	$(2^4)^{\frac{1}{5}} \times 2^x = (2^3)^{\frac{3}{4}}$ $2^{\frac{4}{5}+x} = 2^{\frac{9}{4}}$		This mark is given for a process to use of index laws to derive an equation
	$\dfrac{4}{5} + x = \dfrac{9}{4}$ $x = \dfrac{29}{20} = 1.45$		This mark is given for the correct answer only

Question 8 (Total 3 marks)
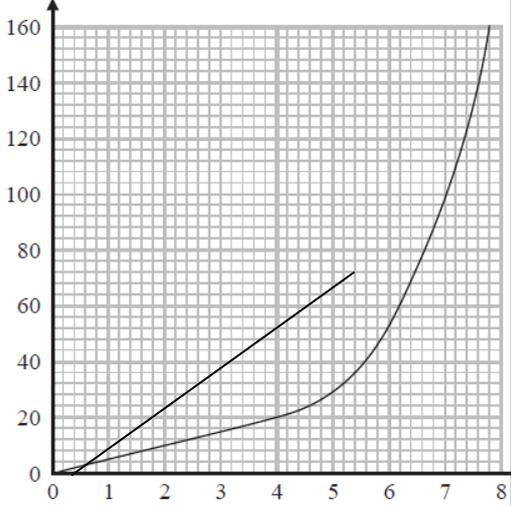
Part	Working or answer an examiner might expect to see	Mark	Notes
		P1	This mark is given for drawing a tangent to the curve at time *t* = 5
	$\dfrac{70-0}{8-3} = \dfrac{70}{5}$	P1	This mark is given for a process to find the gradient
	14 m/s	A1	This mark is given for answer in the range 11 – 19 m/s

Question 9 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes

	Volume shape **A** : volume shape **B** = = (√3)³ : (√4)³ = √27 : 8	M1	This mark is given for a method to find the scale factors of volumes
	Volume shape **A** = $\sqrt{27} \times \dfrac{10}{8}$	M1	This mark is given for a method to use the ratio of volumes to find the volume of shape **A**
	6.5	A1	This mark is given for the correct answer only

Question 10 (Total 2 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\sqrt[4]{\dfrac{16}{81}} = \dfrac{2}{3}$	M1	This mark is given for a method to find the probability of throwing one head
	Probability of getting 4 tails $= \left(1 - \dfrac{2}{3}\right)^4$ $\dfrac{1}{81}$	A1	This mark is given for the correct answer only

Question 11 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	(angle) *BAE* = (angle) *CDE* angles in the same segment are equal or angles at the circumference subtended on the same arc are equal	C1	This mark is given for identifying one pair of equal angles with a correct reason
	(angle) *AEB* = (angle) *DEC* opposite angles or vertically opposite angles are equal	C1	This mark is given for identifying a second pair of equal angles with a correct reason
	Thus the two triangles have three pairs of equal angles and so are similar	C1	This mark is given for a correct conclusion with supporting reasons

Question 12 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	The terms of $2n^2 - 1$ are 1, 7, 17, 31, 49…	M1	This mark is given for a method to generate at least three terms of the first sequence
	The terms of $40 - n^2$ are 39, 36, 31, 24, 15…	M1	This mark is given for a method to generate at least three terms of the second sequence
	31	A1	This mark is given for a correctly identifying the only number in both sequence

Question 13 (Total 2 marks)
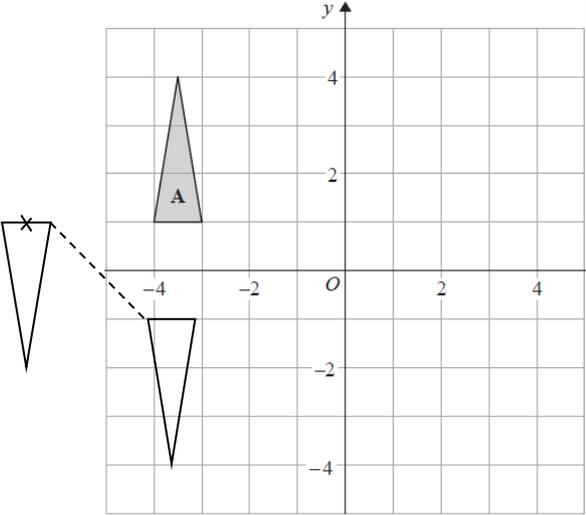
Pa rt	Working an or answer examiner might expect to see	Ma rk	Notes
		M1	This mark is given for showing a rotation of 180° about (−2, 0) followed by a translation $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$
	(−3.5, 1)	A1	This mark is given for the correct answer only

Question 14 (Total 5 marks)

Par t	Working or answer an examiner might expect to see	Mar k	Notes
	$\dfrac{x}{4x-1} = \dfrac{6x+5}{12x+31}$	P1	This mark is given for a process to derive an equation in x
	$(4x-1)(6x+5) = x(12x+31)$	P1	This mark is given for a process to remove fractions from the equation

		$24x^2 + 14x - 5 = 12x^2 + 31x$	P1	This mark is given for a process to form a quadratic equation
		$24x^2 + 14x - 5 - 12x^2 - 31x = 0$		
		$12x^2 - 17x - 5 = 0$		
		$(4x + 1)(3x - 5) = 0$ or $$\dfrac{-(-17) \pm \sqrt{289 - (4 \times 12 \times -5)}}{24}$$ $$= \dfrac{17 \pm \sqrt{529}}{24}$$	P1	This mark is given for a process to solve the quadratic equation, either by factorising or by using the quadratic formula
		$x = \dfrac{5}{3}$ (rejecting $x = -\dfrac{1}{4}$)	A1	This mark is given for the correct answer only

Question 15 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	24 × 4 = 96	P1	This mark is given for a process to find the length of the rectangle
	$48 \sin 60° = 48 \times \dfrac{\sqrt{3}}{2} =$ $24\sqrt{3}$ or $\sqrt{(48^2 - 24^2)} = 24\sqrt{3}$	P1	This mark is given for a process to find the perpendicular height of an equilateral triangle of side 48 cm
	$24 + 24 + 24\sqrt{3} = 89.569…$	P1	This mark is given for a complete process to find the width of rectangle
	8600 (to 3 significant figures)	A1	This mark is given for a correct answer only
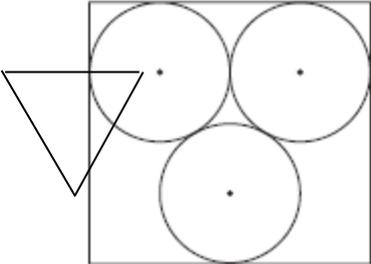
Question 16 (Total 5 marks)

Part	Working an or answer examiner might expect to see	Mark	Notes
	ABC = 360° − 150° − 143° = 67°	P1	This mark is given for a process to find the angle ABC
	$AC^2 = 9^2 + 8^2 − (2 \times 9 \times 8 \times \cos 67°)$ $AC^2 = 88.734…$	P1	This mark is given for a process to use the cosine rule

			$AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \cos ABC$ to find AC
	$AC = 9.412$	P1	This mark is given for correctly finding the distance AC
	$\dfrac{\sin BAC}{9} = \dfrac{\sin 67°}{9.412}$ $\sin BAC = \dfrac{9 \sin 67°}{9.412} =$ 0.8802 $BAC = 61.6°$	P1	This mark is given for a process to use the sine rule to find the angle BAC
	$37 + 62.6 = 098.6$	A1	This mark is given for a correct answer in the range 098.5 to 098.6

Question 17 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$(2x - 1)(10 - x) \times \sin 150°$	M1	This mark is given for a method to find the area of the parallelogram
	$\dfrac{1}{2}(2x - 1)(10 - x) > 15$	M1	This mark is given for a method to form an inequality
	$(2x - 1)(10 - x) > 30$ $-2x^2 + 21x - 10 > 30$ $-2x^2 + 21x - 40 > 0$ $2x^2 - 21x + 40 < 0$	C1	This mark is given for a completely correct chain of reasoning leading to $2x^2 - 21x + 40 < 0$
(b)	$(2x - 5)(x - 8) < 0$	M1	This mark is given for a method to factorise $2x^2 - 21x + 40 < 0$
	$x = 2.5, x = 8$	M1	This mark is given for a method to find the critical values 2.5 and 8

	2.5 < x < 8	A1	This mark is given for the correct range of values for x

Question 18 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\dfrac{1}{2} \times AE \times 24 \sin 30° =$ $6AE$	P1	This mark is given for a process to find the area of one of triangle *AFE* (and thus the area of triangle *BCD*)
	$2 \times 6AE = 12AE$	P1	This mark is given for a process to find the area of the rectangle *ABDE*
	Let $AB = x$ and $AE = 3x$ Then $12 \times 3x = 3x \times x$ $36x = 3x^2$	P1	This mark is given for a process to use the ration *AB*:*AE* = 1: 3 and use the area of the rectangle *ABDE* to find the length *AE*
	$36 = 3x$ $AE = 36$	A1	This mark is given for the correct answer only

Question 19 (Total 4 marks)

Part	Working an or answer examiner might expect to see	Mark	Notes
	$OA = 16 \sin 30° = 8$	P1	This mark is given for a process to find the length OA
	$x^2 + y^2 = 64$	P1	This mark is for a process to use the equation of the circle $x^2 + y^2 = r^2$
	$9p^2 + p^2 = 64$ $10p^2 = 64$ $p = \sqrt{6.4}$	P1	This mark is given for a process to substitute $3p$ and p into the equation for a circle to solve for p
	2.53	A1	This mark is given for a correct answer in the range 2.52 to 2.53

Question 20 (Total 2 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	Rotation 180° or Rotation 90° clockwise or Rotation 90° anticlockwise	C1	This mark is given for finding an angle of the rotation
	about (–1, 2) or (–1, 4) or about (–1, 0) or about (–1, 6)	C1	This mark is given for finding the corresponding centre of rotation

Question 21 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	Gradient $\dfrac{y}{x} = \dfrac{\sqrt{7}}{2} \div \dfrac{3}{2}$ $= \dfrac{\sqrt{7}}{3}$	M1	This mark is given for a method to find gradient of *OP*
	$\dfrac{\sqrt{7}}{3} \times m = -1$, so $m = \dfrac{-3}{\sqrt{7}}$	M1	This mark is given for a method to find gradient *m* of the tangent to *OP*
	Gradient of tangent to $OP, \ y = \dfrac{-3}{\sqrt{7}} x + c$ $\dfrac{\sqrt{7}}{2} = \dfrac{-3}{\sqrt{7}} \times \dfrac{3}{2} + c$ so $c = \dfrac{\sqrt{7}}{2} + \dfrac{9}{2\sqrt{7}} =$ $\dfrac{16}{2\sqrt{7}} = \dfrac{8}{\sqrt{7}}$ Thus equation of the tangent to **L** is $y = \dfrac{-3}{\sqrt{7}} x + \dfrac{8}{\sqrt{7}}$	A1	This mark is given for the correct answer only

Question 22 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$OAD = 90$	B1	This mark is given for a finding the size of the angle *OAD*
	$AOB = 90 + 32 = 122$ $OAB = \dfrac{180 - 122}{2} = 29$	M1	This mark is given for a method to find the size of the angle *OAB*

	$CAB = 180 - 90 - 29 = 61$		A1	This mark is given for the correct answer only

Question 23 (Total 4 marks)
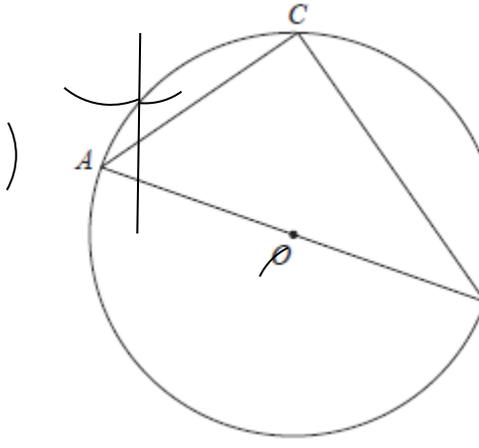
	Working or answer an examiner might expect to see		Notes
	  base angles of an isosceles triangle are equal		This mark is given for drawing the line *OC* to make an isosceles triangles *OBC* and *AOC*
	$x + x + y + y = 180°$ angles in a triangle add up to 180		This mark is given for finding the sum of the angles in triangle *ABC*
	$2x + 2y = 2(x + y) = 180$, so $x + y$ = angle $ACB = 90°$		This mark is given for a complete proof to show ACB = 90°
			This mark is given for a complete proof with all reasons given

Question 1 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$y = \dfrac{17}{2} - \dfrac{3}{2}x,$ gradient $= -\dfrac{3}{2}$	P1	This mark is given for a process to find the gradient of **L**
	Gradient of **M** $= \dfrac{2}{3}$ Equation for **M** is $y = \dfrac{2}{3}x + c$ Point A on the line **M** at $(0, 2) \Rightarrow c = 2$	P1	This mark is given for a process to find an equation for the perpendicular **M**
	$B = (0, \dfrac{17}{2})$	B1	This mark is given for identifying the coordinates of the point B
	$\dfrac{17}{2} - \dfrac{3}{2}x = \dfrac{2}{3}x + 2$ $\dfrac{13}{2} = \dfrac{13}{6}x,$ $x = 3, y = 4$ $C = (3, 4)$	P1	This mark is given finding the coordinates of the point C
	Area $ABC = \dfrac{1}{2}(8.5 - 2) \times 3 = 9.75$	A1	This mark is given for the correct answer only

Question 2 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\dfrac{p-5}{q-5} = \dfrac{5}{1}$; $\dfrac{p+20}{q+20} = \dfrac{5}{2}$	P1	This mark is given for a process to represent the ratios given

	$p - 5 = 5(q - 5)$; $2(p + 20) = 5(q + 20)$	P1	This mark is given for a method to find two simultaneous equations in p and q
	$5q - p = 20$ $5q - 2p = -60$ $p = 80$	M1	This mark is given for a method to rearrange to find a value for p
	$5q = 100$ $q = 20$	M1	This mark is given for a method to find a value for q
	$p : q = 80 : 20$ $= 4 : 1$	A1	This mark is given for the correct answer only

Question 3 (Total 3 marks)

Part	Working an or answer examiner might expect to see	Mark	Notes
	Area for Adley pie chart $= \pi \times 5^2 = 25\pi$ Area of Bridford pie chart $= \pi \times 4^2 = 16\pi$ Angle for 0–19 on Adley = 70°	1	This mark is given for finding areas for the two circles and an angle for 0–19
	$\dfrac{70}{360} \times \dfrac{25\pi}{41\pi}$	1	This mark is given for a method to find the proportion of 0–19 year olds in Bridford
	0.119	1	This mark is given for the correct answer in the range 0.118 – 0.119

Question 4 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes

	1 + 5 = 6 and 7 + 11 = 18 and 6 × 3 = 18		P1	This mark is given for a process to start to solve the problem using multipliers
	(7 + 11) ÷ (1 + 5) = 3 and 1 × 3 : 7 − (3 × 1) : 11		P1	This mark is given for a complete process to find ratios
	3 : 4 : 11		A1	This mark is given for the correct answer only

Question 5 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	Area of triangle $ADC = \frac{1}{2}ab \sin C$ $= 0.5 \times 11 \times (CD \times \sin 105°) = 56$	1	This mark is given for finding an equation for the area of triangle ADC
	$CD = \dfrac{56}{0.5 \times 11 \times \sin 105°} = \dfrac{56}{5.312...}$ $= 10.54$	1	This mark is given for finding the length of CD
	$AC^2 =$ $11^2 + (10.54)^2 - 2 \times 11 \times 10.54 \times \cos 105°$ $= 232.0916 + 60.01496$ $= 292.10656$ $AC = 17.091125$	1	This mark is given for using the cosine rule $c^2 = a^2 + b^2 - 2ab \cos C$ to find the length of AC
	$\dfrac{AB}{\sin 48°} = \dfrac{AC}{\sin 118°}$ $\dfrac{AB}{0.743} = \dfrac{17.09}{0.883}$ $AB = \dfrac{17.09 \times 0.743}{0.883}$	1	This mark is given for a method to us the sine rule to find the length of AB
	14.38	1	This mark is given for an answer in the range 14.3 – 14.4

Question 6 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$g(3) = 3a + b = 20$	1	This mark is given for finding an equation for g(3)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$g(1) = a + b$	1	This mark is given for finding g(1)
	$f^{-1}(x) = \dfrac{x-3}{5}$	1	This mark is given for finding $f^{-1}(x)$
	$f^{-1}(33) = g(1) = a + b = \dfrac{33-3}{5} = 6$	1	This mark is given for finding an the equation $a + b = 6$
	$3a + b = 20$ $a + b = 6$ $2a = 14$, so $a = 7$ and $b = -1$	1	This mark is given for finding the values of a and b

Question 7 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark		Notes
	$3.4^2 = 6.1^2 + 6.2^2 - (2 \times 6.1 \times 6.2 \times \cos C)$	B1		This mark is given for the correct use of the cosine rule $c^2 = a^2 + b^2 \cos C$
	$11.56 = 37.21 + 38.44 - (75.64 \times \cos C)$ $\cos C = \dfrac{64.09}{75.64} = 0.8473\ldots$ $\angle BCA = 38.08°$	P1		This mark is given for a process to find the value of $\angle BCA$
	$\dfrac{DC}{\sin\left(32.08 \times \frac{2}{5}\right)} = $ $\dfrac{6.2}{\sin\left(180 - 32.08 - \left(32.08 \times \frac{2}{5}\right)\right)}$ $\dfrac{DC}{0.222\ldots} = \dfrac{6.2}{0.706\ldots}$		P1	This mark is given for a correct substitution using the sine rule
	$DC = \dfrac{0.222\ldots \times 6.2}{0.706\ldots}$	P1		This mark is given for a process to

Part	Working or answer an examiner might expect to see	Mark	Notes
			find the length *DC*
	1.95	A1	This mark is given for a correct answer only (to 3 significant figures)

Question 8 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
		P1	This mark is given for a process to show some of the information given
		P1	This mark is given for a process to show at least three of the unknown amounts from the information given 5 people speak German and Spanish but not French 3 people speak French and German but not Spanish 22 people speak French but not German or Spanish
		P1	This mark is given for a complete process to find the number of people who speak only Spanish (50 – 44 = 6)
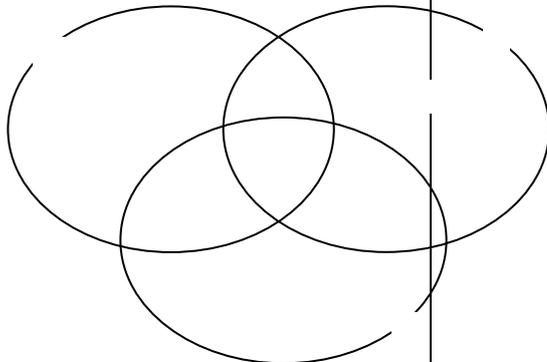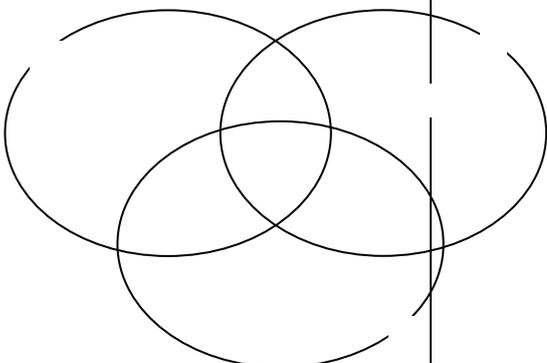
$\dfrac{6}{50} \times \dfrac{6-1}{50-1} = \dfrac{6}{50} \times \dfrac{5}{49}$		P1	This mark is given for a process to find the probability that two people chosen at random speak only Spanish
$\dfrac{6}{490}$		A1	This mark is given for the correct answer (or an equivalent fraction)

Question 9 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	Lengths ratio = $\sqrt[3]{27} : \sqrt[3]{8} = 3 : 2$	1	This mark is for finding a ratio of the lengths associated with the cone
	Areas ratio = $3^2 : 2^2$ = 9 : 4	1	This mark is for finding a ratio of the areas associated with the cone
	Thus the surface area of cone **B** $= \dfrac{297}{9} \times 4 = 132$	1	This mark is given for the correct conclusion following correct arithmetic

Question 10 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$x = 10 + 2y$ $(10 + 2y)^2 + y^2 = 20$	1	This mark is given for a method to start the process by finding a value for x in terms of y and substituting
	$(100 + 20y + 20y + 4y^2) + y^2 = 20$ $4y^2 + 20y + 20y = 100$	1	This mark is given for expanding brackets on the expression obtained
	$5y^2 + 40y + 80 = 0$	1	This mark is given for forming a quadratic equation to be solved
	$(5y + 20)(y + 4) = 0$ $y = -4, x = 2$	1	This mark is given for solving the quadratic equation

|			for y and so find the value of x
	The line intersects the circle at only one point (2, –4), so must be a tangent	1	This mark is given for a fully correct statement to conclude the proof

Question 11 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\dfrac{5}{9} \times \dfrac{4}{8} = \dfrac{20}{72}$	P1	This mark is given for a process to find the probability that the product is an odd number
	$1 - \dfrac{20}{72}$	P1	This mark is given for a process to find the probability that the product is an even number
	$\dfrac{52}{72}$	A1	This mark is given for the correct answer only

Question 12 (Total 2 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$8.3 \leq y < 8.4$	B1	This mark is given for either 8.3 or 8.4 seen in the correct position
		B1	This mark is given for the correct answer only

Question 13 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)		M1	This mark is given for the line $y = x + 4$ drawn
	–0.3 and 3.3	A1	This mark is given for answers in the ranges –0.4 to –0.2 and 3.2 to 3.4
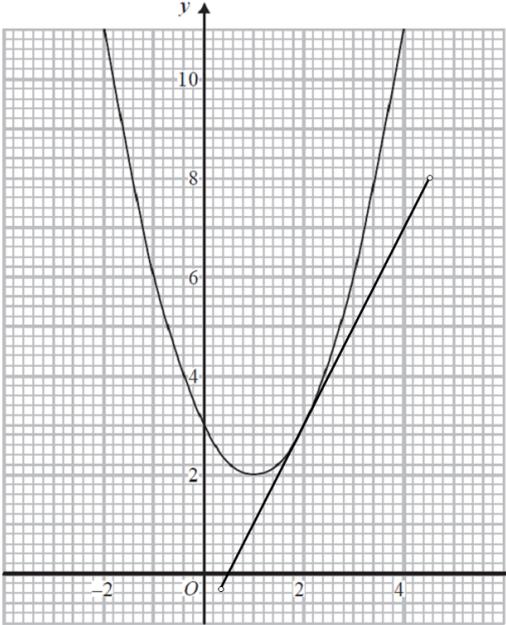
(b)		M1	This mark is given for drawing a tangent to the curve at $x = 2$
	For example, measuring gradient between (2,3) and (4, 7)	M1	This mark is given for a method to find gradient of their tangent
	2	A1	This mark is given for an answer in the range 1.6 to 2.5

Question 14 (Version 1) (Total 5 marks)

Part	Working an or answer examiner might expect to see	Mark	Notes
	$x = \dfrac{7-4y}{3}$	M1	This mark is given for a method to find an expression for x to substitute into $x^2 - 4y^2 = 9$
	$\left(\dfrac{7-4y}{3}\right)^2 - 4y^2 = 9$ $\left(\dfrac{49-56y+16y^2}{9}\right) - 4y^2 = 9$ $49 - 56y + 16y^2 - 36y^2 = 81$	M1	This mark is given for a method to expand and simplify
	$20y^2 + 56y + 32 = 0$ Dividing through by 4 gives $5y^2 + 14y = 8 = 0$	M1	This mark is given for a method to form a quadratic equation to be solved
	$(5y + 4)(y + 2) = 0$	M1	This mark is given factorising the quadratic to solve for y (and hence x)
	$x = 3\dfrac{2}{5}$, $y = -\dfrac{4}{5}$ $x = 5$, $y = -2$	A1	This mark is given for the correct answers only

Question 14 (Version 2) (Total 5 marks)

Part	Working an or answer examiner might expect to see	Mark	Notes
	$y = \dfrac{7-3x}{4}$	M1	This mark is given for a method to find an expression for y to substitute into $x^2 - 4y^2 = 9$

		Mark	Notes
$x^2 - 4\left(\dfrac{49-42x+9x^2}{16}\right)$ $= 9$ $4x^2 - 49 + 42x - 9x^2 = 36$		M1	This mark is given for a method to expand and simplify
$5x^2 - 42x + 85 = 0$		M1	This mark is given for a method to form a quadratic equation to be solved
$(5x - 17)(x - 5) = 0$		M1	This mark is given factorising the quadratic to solve for x (and hence y)
$x = 3\dfrac{2}{5}$, $y = -\dfrac{4}{5}$ $x = 5$, $y = -2$		A1	This mark is given for the correct answers only

Question 15 (Total 5 marks)

Part	Working an or answer examiner might expect to see	Mark	Notes
	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$	M1	This mark is given for finding a vector expression for $\overrightarrow{AB}$
	$\overrightarrow{MN} = -\frac{1}{2}\mathbf{b} + \mathbf{a} + 2\mathbf{a}$ $= -\frac{1}{2}\mathbf{b} + 3\mathbf{a}$	M1	This mark is given for finding a vector expression for $\overrightarrow{MN}$
	$\overrightarrow{PN} = -k(\mathbf{b} - \mathbf{a}) + 2\mathbf{a}$ $= -k\mathbf{b} + (2 + k)\mathbf{a}$	M1	This mark is given for finding a vector expression for $\overrightarrow{PN}$
	Since $\overrightarrow{MN}$ is a multiple of $\overrightarrow{PN}$ $\dfrac{-\frac{1}{2}}{-k} = \dfrac{3}{(2+k)}$ $-\frac{1}{2}(2 + k) = -3k$	M1	This mark is given for recognising that $\overrightarrow{MN}$ is a multiple of $\overrightarrow{PN}$ and comparing coefficients of $\mathbf{a}$ and $\mathbf{b}$

	$k = \dfrac{2}{5}$	A1	This mark is given for the correct answer only

Question 16 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$AD = BC$ (opposite sides of a parallelogram are equal) $\angle BAD = \angle BCD$ (opposite angles of a parallelogram are equal) $\angle ADP = \angle CBQ = 90°$ (given on diagram)	C1	This mark is given for identifying a pair of relevant equal sides or angles with reasons
		C1	This mark is given for a complete identification of all three aspects with reasons given
	Two pairs of angles and one pair of sides are equal (ASA), so *ADP* is congruent to *CBQ*	C1	This mark is given for a correct conclusion of congruency (with reference to ASA)
(b)	$AP = QC$ since triangle *ADP* is congruent to triangle *CBQ*	C1	This mark is given for identifying a pair of equal sides in *APCQ* with reasons
	APCQ is a parallelogram and opposite sides of a parallelogram are equal	C1	This mark is given for a complete explanation

Question 17 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\sqrt{x} + 1 = k \times 1$	1	This mark is given for expressing the common ratio algebraically
	$\dfrac{1}{\sqrt{x} - 1} = \dfrac{\sqrt{x} + 1}{1}$	1	This mark is given for setting up an appropriate equation in x
	$(\sqrt{x} - 1) \times (\sqrt{x} + 1) = 1$ $x - 1 = 1$ $x = 2$	1	This mark is given for the correct answer only
(b)	$(\sqrt{2} + 1) \times (\sqrt{2} + 1)^2$	1	This mark s given for showing that the 5th term = 3rd term $\times$ (common ratio)^2
	$= 2\sqrt{2} + 4 + \sqrt{2} + 2 + 2\sqrt{2} + 1$ $= 7 + 5\sqrt{2}$	1	This mark is given for a correct conclusion supported by working

Question 18 (Total 2 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$7 \leq N < 8$	M1	This mark is given for identifying the numbers 7 and 8
		A1	This mark is given for the correct answer only

Question 19 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	If *CAB* = *x*, then *CRO* = 180 − *x* Opposite angles of a cyclic quadrilateral add up to 180	C1	This mark is given for establishing a relationship between angles in a cyclic quadrilateral with a reason given
	ORB = *x* Angles on a straight line add up to 180	C1	This mark is given for finding the size of angle *ORB* with a reason given
	RO = *OB* since both a radii of the same circle *ABC* = *x* Base angles of an isosceles triangle are equal	C1	This mark is given for finding the size of angle *ABC* with a reason given
	Thus ∠ *CAB* = ∠ *ABC*	C1	This mark is given for a complete proof with all correct reasons given