ANGLO EUROPEAN SCHOOL

IB SL Mathematics: Applications & Interpretation Bridging Work

Examples, Practice Questions & Answers:

10 Bridging Topics to prepare you for IB Maths:

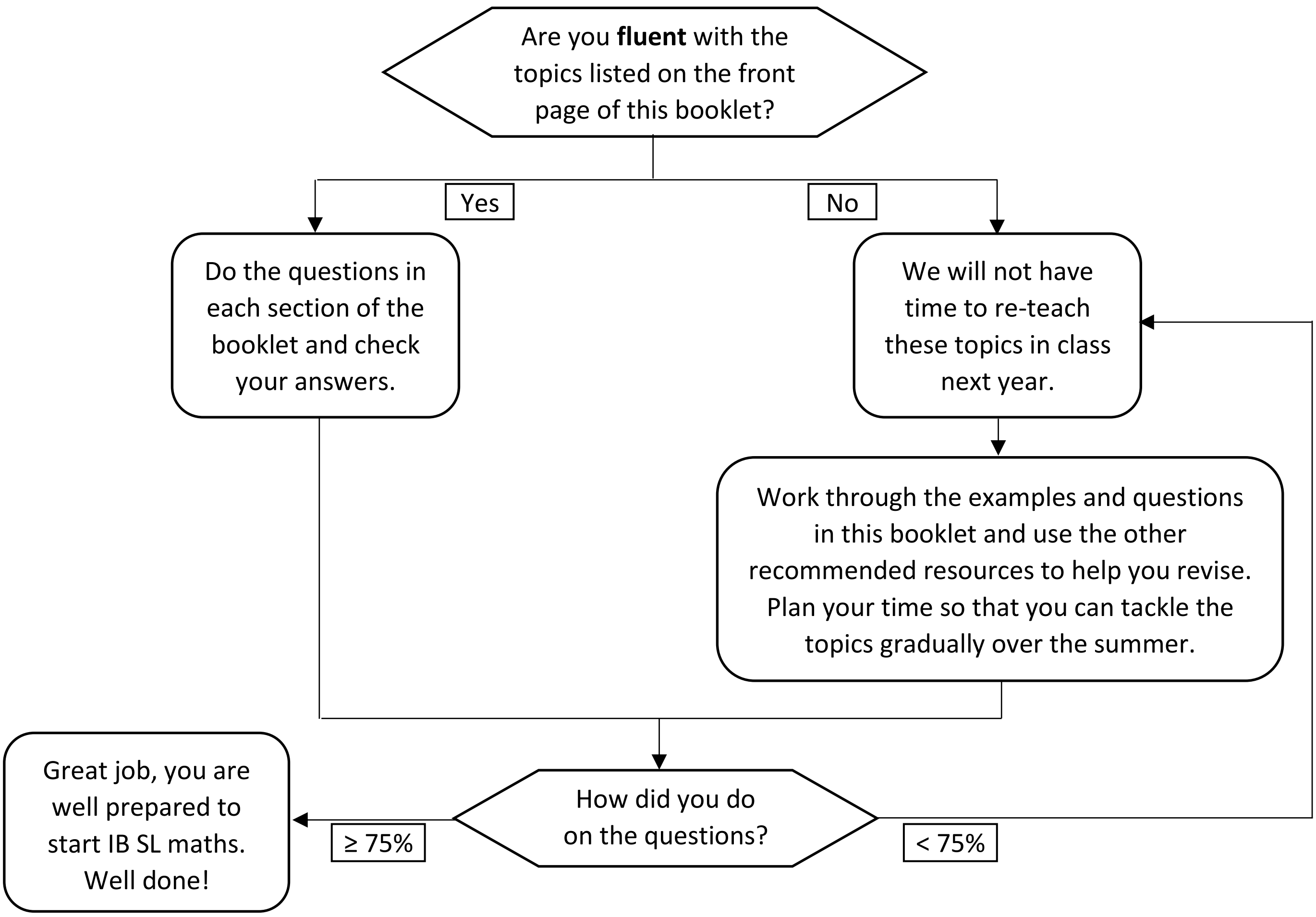
Name: ______________________________

Introduction

Congratulations on choosing to study IB SL Mathematics: Applications and Interpretation. This booklet will help you prepare by brushing up on some of the skills you have learned at GCSE. We will not have time to re-teach these topics in class, but you will need to be **fluent** in them to be able to learn the IB content, so if you don't currently have a good grasp of these topics you need to work on them **NOW** so that you can start with confidence.

Mathematical fluency does not simply mean that you have met this topic before and think that you remember how to do it. To reach fluency, you must be able to **quickly** and **accurately** recall concepts and methods.

There will be a test on the topics from this booklet in the first week of term. It is expected that IB SL students will demonstrate an solid understanding of these topics.



Please bring your completed and self-marked work from this booklet to your first maths lesson in September to show your teacher.

Differences between GCSE and IB SL maths:

GCSE Maths	**IB SL Maths**
You have an exercise book to keep all your work together.	You will need to keep neat, accurate and well-ordered notes and work in your folder.
It's the answer that matters most, but you should show working.	Your course is about applying maths to real-life situations and interpreting the findings. Showing your method is vital to be able to justify your findings. This is great preparation for solving problems in a place of work.
You need to know how to solve problems without using a calculator.	A big part of your course is learning how to use technology (including your graphical calculator) to solve problems and you need your calculator for all of your exams. However, it is extremely beneficial to maintain your basic mental arithmetic skills to allow you to solve problems efficiently as well as to check whether the answer from your calculator is sensible.
How you present your work is not overly important, as long as you get there in the end.	How you present your work can make a big difference to whether anyone can understand your method and therefore have confidence in your findings. If you use technology (e.g. graphical calculator) to find the answer, you must clearly show how you used this.
Your final grade comes only from the final exams.	You will sit 2 final exams at the end of Upper Sixth, but this is only 80% of your final grade. The remaining 20% comes from your Internal Assessment (IA), i.e. coursework. In this piece of work, you will apply the maths you have learned to a project with real-life data; you will need to show and justify your methods in order to reach a convincing conclusion.

Additional website resources:

- Dr Frost – Click 'Practise' and choose 'Practise by Topic' (requires free sign-up if you don't yet have an account) https://www.drfrostmaths.com

- Maths Genie – GCSE questions with model solutions; also has videos. https://www.mathsgenie.co.uk/gcse.html

- Corbett Maths – GCSE questions with model solutions; also has videos. https://corbettmaths.com/contents/

- Exam solutions – videos and GCSE questions with mark schemes https://www.examsolutions.net/gcse-maths/

- Khan academy – linked to American school syllabus, but has very clear videos for many topics https://www.khanacademy.org/math/algebra-home

- Mathswatch – interactive GCSE questions and videos. If you studied with us for Year 11 you will already have an account; click 'Videos' and search for the topic you wish to study then click 'Interactive questions' https://vle.mathswatch.co.uk/vle/

Section 1

Video links:

1) https://www.mathsgenie.co.uk/negativenumbers.php
2) https://www.mathsgenie.co.uk/substitution.php

Calculating and substituting with negative numbers

Key points

- When a negative number is added to a starting value, the answer is always smaller than the starting value, i.e. adding a negative is equivalent to subtraction.
  Examples:
  a) 8 + (-3) = 8 – 3 = 5
  b) (-2) + (-5) = (-2) – 5 = -7
  c) (-18) + (-7) = (-18) – 7 = -25

- When a negative number is subtracted from a starting value, the answer is always greater than the starting value, i.e. subtracting a negative is equivalent to addition.
  Examples:
  a) 8 – (-3) = 8 + 3 = 11
  b) (-2) – (-5) = (-2) + 5 = 3
  c) (-18) – (-7) = (-18) + 7 = -11

- When a negative number is multiplied/divided by a positive number (or a positive number is multiplied/divided by a negative number), the answer is always negative.
  Examples:
  a) 3 x (-4) = -12
  b) (-5) x 2 = -10
  c) (-12) ÷ 6 = -2
  d) 14 ÷ (-2) = -7

- When two negative numbers are multiplied/divided, the answer is always positive.
  Examples:
  a) (-6) x (-3) = 18
  b) $(-3)^2$ = (-3) x (-3) = 9
  c) (-6) ÷ (-3) = 2

- When substituting into algebraic expressions, remember that terms without an operation between them are always **multiplied**.
  Examples:
  a) If a = -7 and b = 13, evaluate: 5ab

$5ab = 5 \times (-7) \times 13$ $= (-35) \times 13$ $= -455$	**1** 5ab means 5 × a × b **2** A positive and negative multiplied makes a negative answer.

b) If $x = -5$, $y = -8$ and $z = -3$, evaluate: $x^2 + 2y - 6z$

$x^2 + 2y - 6z = (-5)^2 + 2 \times (-8) - 6 \times (-3)$ $= 25 + (-16) - (-18)$ $= 25 - 16 + 18$ $= 27$	**1** $2y$ means $2 \times y$ and $6z$ means $6 \times z$ **2** Two negative numbers multiply to make a positive and a positive and negative multiplied makes a negative. **3** Adding a negative number is equivalent to subtraction and subtracting a negative number is equivalent to addition.

c) If $f = -2$, $g = -6$ and $h = -10$, evaluate: $fg(g - h \div f)$

$fg(g - h \div f) = (-2) \times (-6) \times \big((-6) - (-10) \div (-2)\big)$ $= (-2) \times (-6) \times \big((-6) - 5\big)$ $= (-2) \times (-6) \times \big(-11\big)$ $= 12 \times \big(-11\big)$ $= -132$	**1** fg means $f \times g$ and $fg(g$… means $fg \times (g$… **2** Remember to complete the calculation in the correct order. **3** Two negative numbers multiply/divide to make a positive and a positive and negative multiplied makes a negative.

Exercise 1

1. When a = -5, b = 3 and c = -2 find the value for the expressions.

a) 5a
b) 5a + 10c
c) 2(a + c)
d) c(5 + a)
e) 9(b – c)
f) ab – c
g) a^2 + b
h) -2(a^2 + b)

2. Use the formula y = 4f – 7 to complete the questions.
a) Find the value of y when f = -2

b) Find the value of f when y = -17

3. Use the formula X = -3(a^2 + b) to complete the questions.
a) Find the value of X when a = -4 and b = 8
b) Find the value of a when X = -60, and b = -5

ANSWERS 1

1. When a = -5, b = 3 and c = -2 find the value for the expressions.

a)	5a	-25
b)	5a + 10c	-45
c)	2(a + c)	-14
d)	c(5 + a)	0
e)	9(b – c)	45
f)	ab – c	-13
g)	$a^2 + b$	28
h)	$-2(a^2 + b)$	-56

2. Use the formula y = 4f – 7 to complete the questions.

a) Find the value of y when f = -2

y = -15

b) Find the value of r when y = -17

f = -2.5

3. Use the formula $X = -3(a^2 + b)$ to complete the questions.

a) Find the value of X when a = -4 and b = 8 X = -72

b) Find the value of a when X = -60, and b = -5 a = 5 or -5

Section 2a

Video links:

1) https://corbettmaths.com/2012/08/21/expressing-one-quantity-as-a-percentage-of-another/
2) https://www.mathsgenie.co.uk/percentages.php

Writing percentages and finding percentages

Key points

- 'Per cent' means 'out of 100', so a percentage is always a measure out of 100. This means it is easy to write a percentage as a fraction, e.g. 59% = $\frac{59}{100}$

- To write a fraction as a percentage, if possible, first create an equivalent fraction with a denominator of 100, e.g. 13 out of 20 = $\frac{13}{20} = \frac{65}{100}$ = 65%.

- If it is not possible to create an equivalent fraction with a denominator of 100, then first convert the fraction to a decimal by dividing the numerator by the denominator, then multiply this by 100 to make the percentage, e.g. 5 out of 8 = 5 ÷ 8 = 0.625 = 62.5% (x 100)

- To find a percentage of an amount you can use a number of different methods:
    1. Find the equivalent fraction of the amount
       e.g. find 40% of 230
       40% = $\frac{40}{100} = \frac{2}{5}$
       So, find $\frac{2}{5}$ of 230
       230 ÷ 5 x 2 = 92
       i.e. 40% of 230 = 92

    2. Multiply the amount by the equivalent decimal
       e.g. find 32% of 45
       32% = 0.32
       So, calculate 45 x 0.32
       = 14.4
       i.e. 32% of 45 = 14.4

    3. Use a combination of percentages to 'build' the required percentage
       e.g. find 76% of 860
       76% = 25% x 3 + 1%
       25% of 860 = $\frac{1}{4}$ of 860 = 860 ÷ 4 = 215
       1% of 860 = $\frac{1}{100}$ of 860 = 860 ÷ 100 = 8.6
       So, 76% of 860 = 215 x 3 + 8.6
       = 653.6

Exercise 2a

1) Using any method you prefer, work out the following.
Give your answer to 2 d.p. where appropriate.

a) 72 as a percentage of 200

b) 24 as a percentage of 50

c) 19 as a percentage of 24

d) 7 as a percentage of 8

2) Here is a table of sales from a clothes store in a week.

The manager wants to find out the percentage of returns. Which of the items has the highest percentage of returns compared to the sales of that item?

Type	Number of sales	Number of returns
Dresses	23	12
Trousers	11	6
Hats	6	3
T-shirts	64	33

3) Hannah is paid £280.
She spends 30% on her rent, 25% on food and bills and saves the rest.

(a) How much does Hannah spend on rent?
(b) How much does Hannah spend on food and bills?
(c) How much does Hannah save?

4) Maria has invested different amounts of money in 3 different companies. Over the past year, her investments have increased in value by the percentages shown in the table below.

Which investment has earned Maria the most money over the past year?

Company	Percentage gain	Amount invested
Pineapple	2.5%	£240
Tech smart	8.1%	£60
Coffee world	4.2%	£150

ANSWERS 2a

1) a) $\frac{72}{200}$ = 36%

b) $\frac{24}{50}$ = 48%

c) $\frac{19}{24}$ = 79.1666…%

d) $\frac{7}{8}$ = 87.5%

2) Dresses = $\frac{12}{23}$ = 52.173…%

Trousers = $\frac{6}{11}$ = 54.545…%

Hats = $\frac{3}{6}$ = 50%

T-shirts = $\frac{33}{64}$ = 51.5625%

Trousers have the highest percentage of returns compared to sales.

3) a) 30% of £280 = £84

b) 25% of £280 = £70

c) £280 – £84 – £70 = £126

4) Pineapple = 2.5% of £240 = £6

Tech smart = 8.1% of £60 = £4.86

Coffee world = 4.2% of £150 = £6.30

Coffee world has made Maria the most money over the past year.

Section 2b

Video links:

1) https://corbettmaths.com/2012/08/21/increasing-or-decreasing-by-a-percentage/
2) https://www.youtube.com/watch?v=6Pbzq322RzU

Percentage increase/decrease

Key points

- To increase/decrease an amount by a percentage you can use two different methods:
    1. Find the required percentage of the amount then add/subtract this from the original amount

       e.g.1) increase 80 by 23%

       23% of 80 = 18.4

       80 + 18.4 = 98.4

       i.e. increase 80 by 23% = 98.4

       e.g.2) decrease 80 by 23%

       23% of 80 = 18.4

       80 – 18.4 = 61.6

       i.e. decrease 80 by 23% = 61.6

    2. Work out what the final percentage will be then find this percentage of the amount (remember that the original amount is always worth 100%)

       e.g.1) increase 72 by 35%

       100% + 35% = 135%

       135% of 72 = 97.2

       i.e. increase 72 by 35% = 97.2

       e.g.2) decrease 72 by 35%

       100% – 35% = 65%

       65% of 72 = 46.8

       i.e. decrease 72 by 35% = 46.8

Exercise 2b

1) Calculate the following:

a) Increase 4584 by 24%

b) Decrease 3815 by 65%

c) Increase 763 by 2.5%

d) Decrease 850 by 0.48%

2) A mathematician has her salary of £2000 per month increased by 6%.

60% of her new salary pays for bills, rent, etc.

$\frac{1}{4}$ of her new salary is spent on food.

She donates 2% of her new salary to charity.

How much does she have left per month for savings?

3) Zara wants to buy 72 candles.
Each candle costs £4.80

There is a special offer

Work out the cost of buying 72 candles using the special offer.

Special Offer
Candles £4.80 each
Buy 60 or more candles and get 15% off the total cost.

4) An empty flowerpot has a mass of 800g.
The mass of the flowerpot increases to 4kg when filled with soil.

A different flowerpot is 25% lighter but holds 40% more soil.
Calculate the mass of this flowerpot when it is full of soil.

ANSWERS 2b

1) a) 4584 x 1.24 = 5684.16

b) 3815 x 0.35 = 1335.25

c) 763 x 1.025 = 782.075

d) 850 x 0.9952 = 845.92

2) Increase £2000 by 6% = 2000 x 1.06 = £2120

60% of £2120 = £1272

$\frac{1}{4}$ of £2120 = £530

2% of £2120 = £42.40

£2120 – £1272 – £530 – £42.40 = £275.60

£275.60 is left for the mathematician to save

3) Full price: 72 candles = 72 x £4.80 = £345.60

Decrease £345.60 by 15% = £293.76

Using the special offer, 72 candles costs £293.76

4) Mass of soil = 4 kg – 800 g = 3.2 kg

Mass of 2^{nd} flowerpot: decrease 800 g by 25% = 600 g

Mass of soil for 2^{nd} flowerpot: increase 3.2 kg by 40% = 4.48 kg

Mass of 2^{nd} flowerpot when full of soil = 600 g + 4.48 kg = 5.08 kg *or* 5080 g

Section 3

Video link:
https://corbettmaths.com/2013/12/23/changing-the-subject-video-7/

Rearranging equations

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.

Examples

Example 1 Make b the subject of the formula $a = b - 8$

$a = b - 8$ $a + 8 = b$ $b = a + 8$	**1** All the terms containing *b* are already on one side and everything else is on the other side. **2** Add 8 to both sides of the formula. **3** You could swap the sides so the subject comes first (you don't *have* to).

Example 2 Make *h* the subject of the formula $k = 3h$

$k = 3h$ $\frac{k}{3} = h$ $h = \frac{k}{3}$	**1** All the terms containing *h* are already on one side and everything else is on the other side. **2** Divide throughout by 3. **3** You could swap the sides so the subject comes first (you don't *have* to)

Example 3 Make *t* the subject of the formula $v = u + at$.

$v = u + at$ $v - u = at$ $\frac{v-u}{a} = t$ $t = \frac{v-u}{a}$	**1** Get the terms containing *t* on one side and everything else on the other side. **2** Divide throughout by *a*. **3** You could swap the sides so the subject comes first (you don't *have* to).

Example 4 Make n the subject of the formula $S = \frac{n}{2}(a + l)$

$S = \frac{n}{2}(a + l)$	
$2S = n(a + l)$	**1** Multiply throughout by 2.
$\frac{2S}{a+l} = n$	**2** Divide throughout by $(a + l)$.
$n = \frac{2S}{a+l}$	**3** You could swap the sides so the subject comes first (you don't *have* to).

Exercise 3

Change the subject of each formula to the letter given in the brackets.

1 $k = m + g$ [g] **2** $C = \pi d$ [d] **3** $P = 2l + 2w$ [w]

4 $x = a + (n - 1)d$ [a] **5** $D = \dfrac{S}{T}$ [T] **6** $x = \dfrac{b - c}{d}$ [b]

7 **a** Make A the subject in the formula $\cos(\theta) = \frac{A}{H}$

b Make H the subject of the formula $\cos(\theta) = \frac{A}{H}$

8 Make r the subject of the following formulae.

a $C = 2\pi r$ **b** $A = \pi r^2$ **c** $V = \dfrac{4}{3}\pi r^3$ **d** $V = \dfrac{2}{3}\pi r^2 h$

ANSWERS 3

1 $g = k - m$

2 $d = \dfrac{C}{\pi}$

3 $w = \dfrac{P - 2l}{2}$

4 $a = x - (n - 1)d$

5 $T = \dfrac{S}{D}$

6 $b = dx + c$

7 **a** $A = H \times \cos(\theta)$

b $H = \frac{A}{\cos(\theta)}$

8 **a** $r = \frac{C}{2\pi}$

b $r = \sqrt{\frac{A}{\pi}}$

c $r = \sqrt[3]{\frac{3V}{4\pi}}$

d $r = \sqrt{\frac{3V}{2\pi h}}$

Section 4

Video links:

1) https://corbettmaths.com/2013/12/23/expanding-brackets-video-13/
2) https://corbettmaths.com/2013/12/23/expanding-two-brackets-video-14/

Expanding brackets and simplifying expressions

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket

Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$ $= 3 - 5x$	**1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4 **2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$

Example 3 Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$ $= x^2 + 2x + 3x + 6$ $= x^2 + 5x + 6$	**1** Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3 **2** Simplify by collecting like terms: $2x + 3x = 5x$

Example 4 Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$ $= 2x^2 + 3x - 10x - 15$ $= 2x^2 - 7x - 15$	**1** Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5 **2** Simplify by collecting like terms: $3x - 10x = -7x$

Exercise 4

> **Watch out!**
>
> When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is '+'; if the signs are different the answer is '–'.

1 Expand.

a $3(2x - 1)$ **b** $-2(5pq + 4q^2)$

2 Expand and simplify.

a $7(3x + 5) + 6(2x - 8)$ **b** $8(5p - 2) - 3(4p + 9)$

3 Expand.

a $3x(4x + 8)$ **b** $-2h(6h^2 + 11h - 5)$

4 Expand and simplify.

a $3(y^2 - 8) - 4(y^2 - 5)$ **b** $2x(x + 5) + 3x(x - 7)$

5* Expand $\frac{1}{2}(2y - 8)$

6* Expand and simplify.

a $(x + 4)(x + 5)$ **b** $(x + 7)(x - 2)$

c $(x + 5)(x - 5)$ **d** $(2x + 3)(x - 1)$

e $(5x - 3)(2x - 5)$ **f** $(x + 5)^2$

g $(2x - 7)^2$

Extension

7* Expand and simplify $(x + 3)^2 + (x - 4)^2$

ANSWERS 4

1 **a** $6x - 3$ **b** $-10pq - 8q^2$

2 **a** $21x + 35 + 12x - 48 = 33x - 13$

b $40p - 16 - 12p - 27 = 28p - 43$

3 **a** $12x^2 + 24x$ **b** $10h - 12h^3 - 22h^2$

4 **a** $-y^2 - 4$ **b** $5x^2 - 11x$

5 $y - 4$

6 **a** $x^2 + 9x + 20$ **b** $x^2 + 5x - 14$

c $x^2 - 25$ **d** $2x^2 + x - 3$

e $10x^2 - 31x + 15$ **f** $x^2 + 10x + 25$

g $4x^2 - 28x + 49$

7 $2x^2 - 2x + 25$

Section 5a

Video links:

1) https://www.mathsgenie.co.uk/solving-equations.php
2) https://corbettmaths.com/2012/08/24/solving-equations-with-letters-on-both-sides/

Solve linear equations

Key points

- To solve an equation means to find the value of the variable that makes the equation true.
- To solve an equation, use inverse operations to simplify the equation until you have the value of the variable.

Examples

Example 1 Solve $2x - 5 = 7$

$2x - 5 = 7$ $2x = 12$ $x = 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 2 Solve $2 - 5x = -8$

$2 - 5x = -8$ $-5x = -10$ $x = 2$	**1** Subtract 2 from both sides. **2** Divide both sides by -5.

Example 3 Solve $4(x - 2) = 3(9 - x)$

$4(x - 2) = 3(9 - x)$ $4x - 8 = 27 - 3x$ $7x - 8 = 27$ $7x = 35$ $x = 5$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

Exercise 5a

1 Solve these equations.

a $x + 47 = 31$ **b** $8x - 7 = 3$ **c** $13 = 5 - \frac{x}{4}$

d $\frac{3x}{5} + 2 = 2.9$ **e** $\frac{4x+5}{3} = 6$ **f** $3x + 4(5x - 7) = -120$

2 Solve these equations.

a $9x - 10 = 7x + 24$ **b** $34 - 12x = 28x - 36$

c $5(2x + 9) - 2(x + 11) = 3(3x + 4) + 46$

3 A rectangular field has a perimeter of 150 m.
The field is 12 metres longer than it is wide.
The width of the field is *w* metres.

a Write down an equation, in terms of *w*, to show this information
b Solve your equation to find the width of the field
c Calculate the area of the field.

4 Sam thinks of a number, he calls it *x*.
Sam adds 3 to his number, then multiplies the result by 6.
The answer is now 8 times his original number.

a Write down an equation, in terms of *x*, to show this information
b Solve your equation to find the number Sam thought of.

ANSWERS 5a

1	**a**	$x = -16$	**b**	$x = 1.25$	**c**	$x = -32$
	d	$x = 1.5$	**e**	$x = 3.25$	**f**	$x = -4$

2	**a**	$x = 17$	**b**	$x = 1.75$	**c**	$x = -35$

3 **a** $4w + 24 = 150$

b $w = 31.5$

The field is 31.5 metres wide

c $A = 31.5 \times (31.5 + 12)$

$= 1370.25$

The area of the field is 1370.25 m^2

4 **a** $6(x + 3) = 8x$

b $x = 9$

The number Sam thought of was 9.

Section 5b

Video links:
1) https://corbettmaths.com/2013/05/07/solving-inequalities-one-sign-corbettmaths/
2) https://corbettmaths.com/2013/05/12/solving-inequalities-two-signs/

Solve linear inequalities

A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

Example 1 Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 2 Solve $2 - 5x \geq -8$

$2 - 5x \geq -8$ $-5x \geq -10$ $x \leq 2$	**1** Subtract 2 from both sides. **2** Divide both sides by −5. Remember to reverse the inequality when dividing by a negative number.

Example 3 Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$ $x > 5$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

Exercise 5b

1 Solve these inequalities.

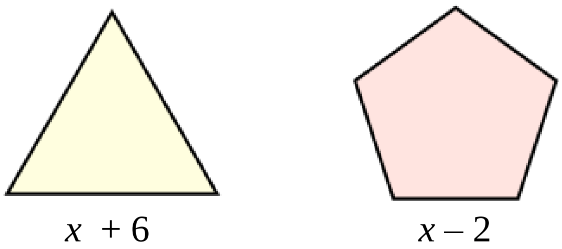
a $4x > 16$ **b** $5x - 7 \leq 3$ **c** $1 \geq 3x + 4$

d $5 - 2x < 12$ **e** $\frac{x}{2} \geq 5$ **f** $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

a $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$ **c** $3(2 - x) > 2(4 - x) + 4$

3 The shapes shown are both **regular**.
The length of one of their sides is shown, in terms of *x*.
The perimeter of the pentagon is greater than the perimeter of the triangle.



a Write down an inequality, in terms of *x* to show this information

b Solve your inequality to find the possible range of values for *x*.

Extension

4 Find the range of values of *x* which satisfies **both**

$$3(x + 2) \leq 30 \quad \textit{and} \quad 4x + 3 > 21$$

ANSWERS 5b

1	**a**	$x > 4$	**b**	$x \leq 2$	**c**	$x \leq -1$
	d	$x > -3.5$	**e**	$x \geq 10$	**f**	$x < -15$
2	**a**	$t < 2.5$	**b**	$n \geq 1.4$	**c**	$x < -6$
3	**a**	$3(x + 6) < 5(x - 2)$	**b**	$x > 14$		

4 $3(x + 2) \leq 30$ $4x + 3 > 21$

$x \leq 8$ $x > 4.5$

i.e. $4.5 < x \leq 8$

Section 6a

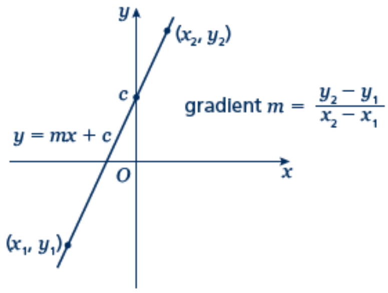
Video links:

1) https://corbettmaths.com/2013/05/29/y-equals-graphs/
2) https://corbettmaths.com/2013/05/29/x-equals-graphs/
3) https://corbettmaths.com/2013/05/29/finding-the-equation-of-a-straight-line/

Straight line graphs

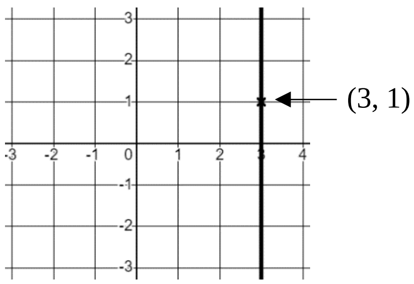
Key points

- A horizontal line has the equation $y = a$, where a is the y-intercept.
- A vertical line has the equation $x = b$, where b is the x-intercept.
- A diagonal line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept.
- The gradient of a line describes how steep it is. If the gradient is negative the line slopes downwards, if the gradient is positive the line slopes upwards (from left-right). The value of the gradient tells you the vertical change if the x-value is increased by 1.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

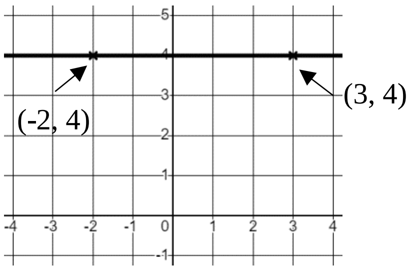


Examples

Example 1 A vertical line passes through the point (3, 1). Write the equation of the line.

 x-intercept = 3 So x = 3	**1** A vertical line has equation $x = b$, where b is the x-intercept.

Example 2 A straight line passes through the points (-2, 4) and (3, 4). Write the equation of the line.

 Line is horizontal y-intercept = 4 So $y = 4$	**1** A horizontal line has equation $y = a$, where a is the y-intercept.

Example 3 A straight line has gradient $-\frac{1}{2}$ and y-intercept 3.

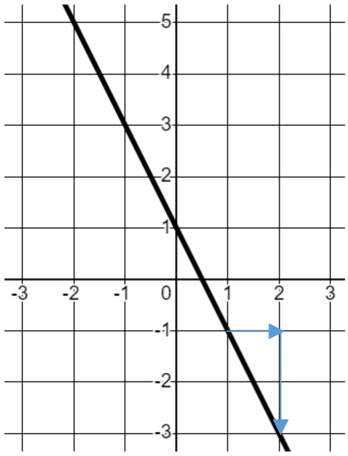
Write the equation of the line in the form $y = mx + c$.

$m = -\frac{1}{2}$ and $c = 3$ So $y = -\frac{1}{2}x + 3$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation.

Example 4 Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ Gradient = $m = \frac{2}{3}$ y-intercept = $c = -\frac{4}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \ldots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 5 Find the equation of the line shown.



y-intercept is 1 So $c = 1$	**1** Find the y-intercept of the line
Gradient is -2 So $m = -2$	**2** Find the gradient of the line
So $y = -2x + 1$	**3** Substitute $c = 1$ and $m = -2$ into the equation $y = mx + c$

Exercise 6a

1 Find the gradient and the *y*-intercept of the following equations.

a $y = 3x + 5$ **b** $y = -\frac{1}{2}x - 7$

c $2y = 4x - 3$ **d** $2x - 3y - 7 = 0$

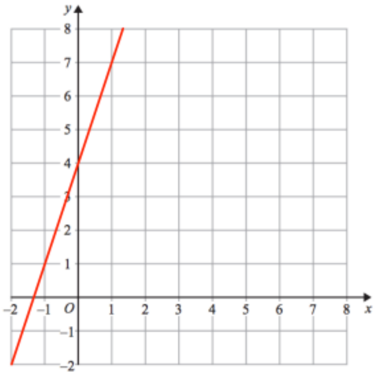
Hint
Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

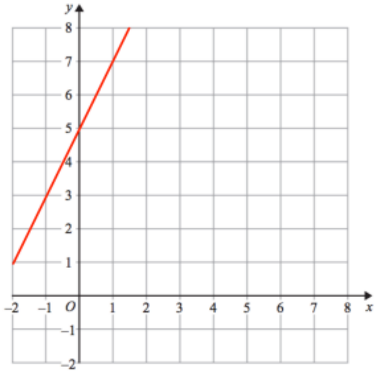
Gradient	*y*-intercept	Equation of the line
5	0	
–3	2	
4	–7	

3 Write the equation of each of the following lines

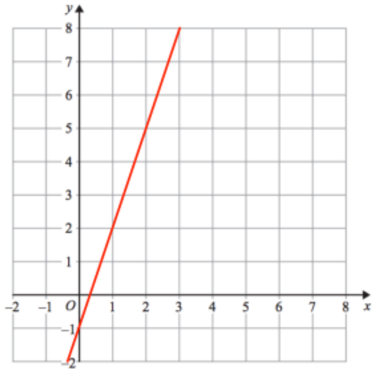
(a)



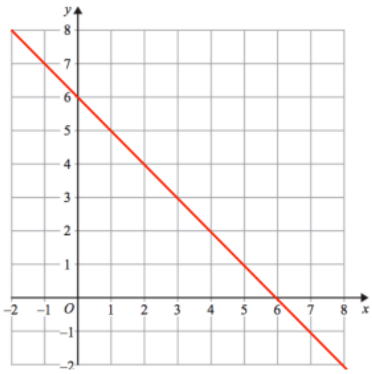
(b)



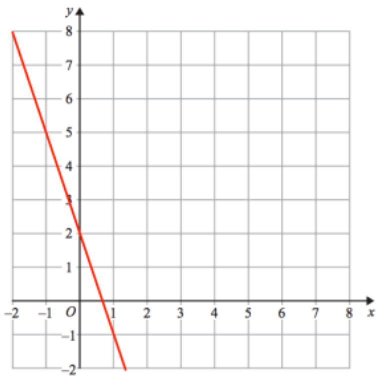
(c)



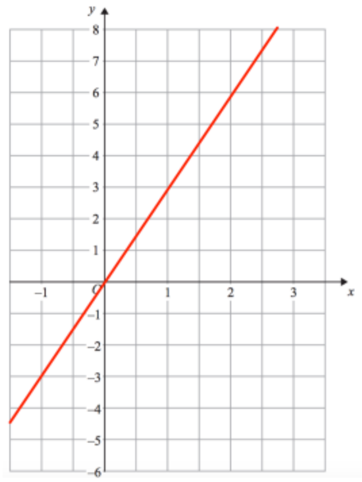
(d)



(e)

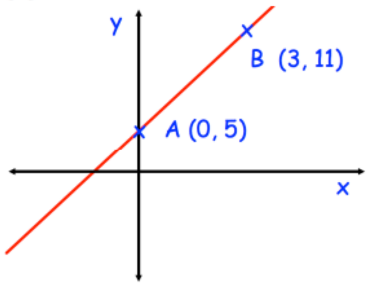


(f)

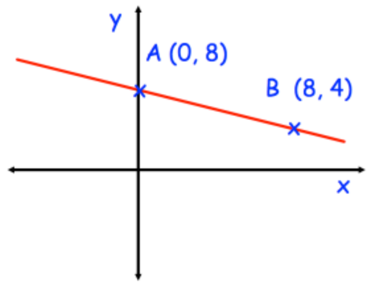


4 Write the equation of each of the following lines

(a)



(b)



ANSWERS 6a

1 **a** $m = 3, c = 5$ **b** $m = -\frac{1}{2}, c = -7$

c $m = 2, c = -\frac{3}{2}$ **d** $m = \frac{2}{3}, c = -\frac{7}{3}$ or $-2\frac{1}{3}$

2

Gradient	*y*-intercept	Equation of the line
5	0	$y = 5x$
–3	2	$y = -3x + 2$
4	–7	$y = 4x - 7$

3 **a** $y = 3x + 4$ **b** $y = 2x + 5$ **c** $y = 3x - 1$

e $y = -x + 6$ **f** $y = -3x + 2$ **g** $y = 1.5x$

4 **a** $y = 3x + 5$ **b** $y = -0.5x + 8$

Section 6b

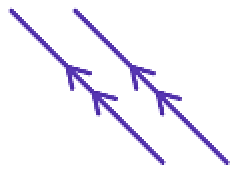
Video links:
1) https://corbettmaths.com/2013/06/06/graphs-parallel-lines/
2) https://corbettmaths.com/2013/06/06/perpendicular-lines-2/

Parallel and perpendicular lines

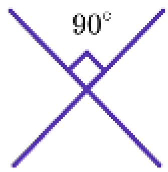
A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- If two lines are parallel, they have the same gradient.
- If two lines are perpendicular, the product of their gradients is -1. This means that the two gradients are the negative reciprocals of each other, i.e. if the gradient of one of the lines is *m* then the gradient of the second line is $\frac{-1}{m}$.



Parallel Lines



Perpendicular Lines

Examples

Example 1 Write down the equation of the line that is parallel to $y = 3x + 1$ and passes through the point (0, 7)

$y = 3x + 1$	**1** The gradient of this line is 3.
(0, 7) is on the *y*-axis	**2** The y-intercept of the line is 7.
$y = 3x + 7$	**3** In the form $y = mx + c$, the gradient is *m* and the *y*-intercept is *c*.

Example 2 Write down the equation of the line that is parallel to $2y + 8x = 14$ and passes through the point (0, -3)

$2y + 8x = 14$	**1** Rearrange into the form $y = mx + c$
$y = - 4x + 7$	**2** The gradient of the line is - 4.
(0, -3) is on the *y*-axis	**3** The y-intercept of the line is - 3.
$y = - 4x – 3$	**4** In the form $y = mx + c$, the gradient is *m* and the *y*-intercept is *c*.

Example 3 Write down the equation of the line that is perpendicular to $y = 3x + 1$ and passes through the point (0, 5)

$y = 3x + 1$	**1** The gradient of this line is 3.
Gradient = $-\frac{1}{3}$	**2** The gradient of the required line is the negative reciprocal of 3.
(0, 5) is on the *y*-axis	**3** The y-intercept of the line is 5.
$y = -\frac{1}{3}x + 5$	**3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 4 Write down the equation of the line that is perpendicular to $y = 0.5x - 5$ and passes through the point (0, 8)

$y = 0.5x - 5$	**1** The gradient of this line is 0.5 or $\frac{1}{2}$.
Gradient = - 2	**2** The gradient of the required line is the negative reciprocal of $\frac{1}{2}$.
(0, 8) is on the *y*-axis	**3** The y-intercept of the line is 8.
$y = -2x + 5$	**4** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 5 Write down the equation of the line that is perpendicular to $y = -3.5x + 2$ and passes through the point (0, - 8)

$y = -3.5x + 2$	**1** The gradient of this line is – 3.5 or $-\frac{7}{2}$.
Gradient = $\frac{2}{7}$	**2** The gradient of the required line is the negative reciprocal of $-\frac{7}{2}$.
(0, - 8) is on the *y*-axis	**3** The y-intercept of the line is - 8.
$y = \frac{2}{7}x - 8$	**4** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Exercise 6b

1 Write down the equation of each of the following lines

a parallel to $y = 3x - 5$ and passing through (0, 2)

b parallel to $y = -5x - 8$ and passing through (0, 6)

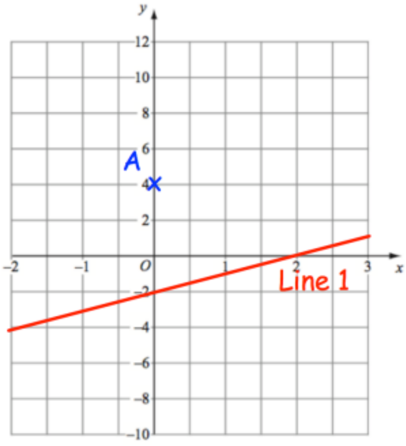
c parallel to $y = -0.5x + 10$ and passing through the origin

d parallel to $x + y = 2$ and passing through (0, - 4)

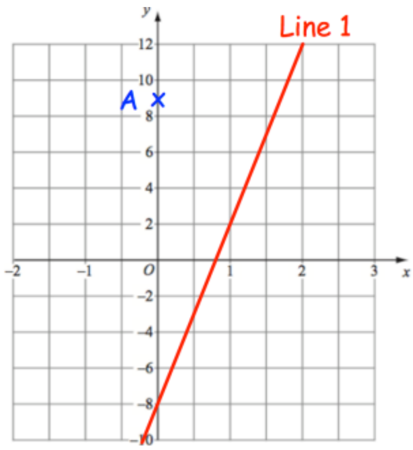
e parallel to $x - 2y + 3 = 0$ and passing through (0, 5)

2 Write down the equation of the line parallel to line 1 and passing through point A

a



b



HINT: Check the scales on the axes carefully

3 Write down the equation of each of the following lines

a perpendicular to $y = -3x - 8$ and passing through (0, 2)

b perpendicular to $y = \frac{1}{3}x + 5$ and passing through the origin

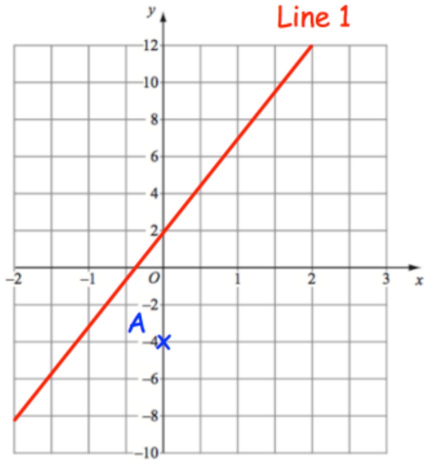
c perpendicular to $y = -\frac{2}{9}x - 1$ and passing through (0, - 4)

d perpendicular to $y = 2\frac{3}{4}x + 9$ and passing through (0, 3)

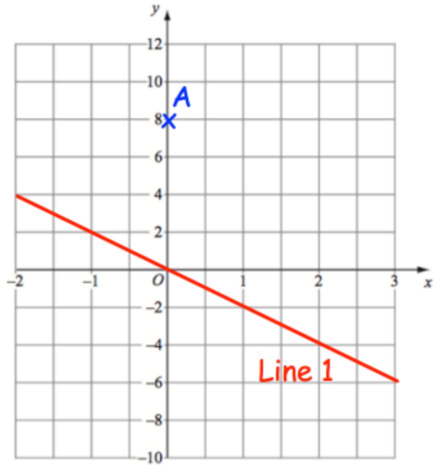
e perpendicular to $5x - 3y + 3 = 0$ and passing through (0, -1)

4 Write down the equation of the line perpendicular to line 1 and passing through point A

a



b



HINT: Check the scales on the axes carefully

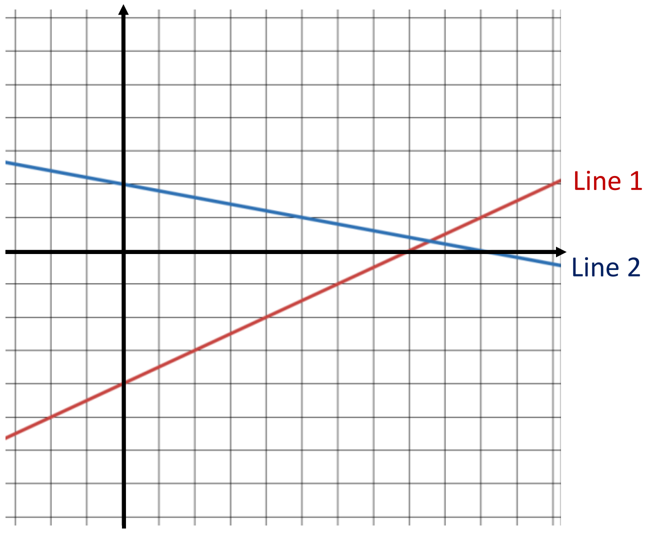
Extension

5 Two straight lines are shown.

Line 1 has equation $y = \frac{3}{2}x - 24$

a Find the equation of Line 2

b Are the lines perpendicular? Show your working to justify your answer.



ANSWERS 6b

1 **a** $y = 3x + 2$ **b** $y = -5x + 6$

c $y = -0.5x$ **d** $y = -x - 4$

e $y = 0.5x + 5$

2 **a** $y = x + 4$ **b** $y = 10x + 9$

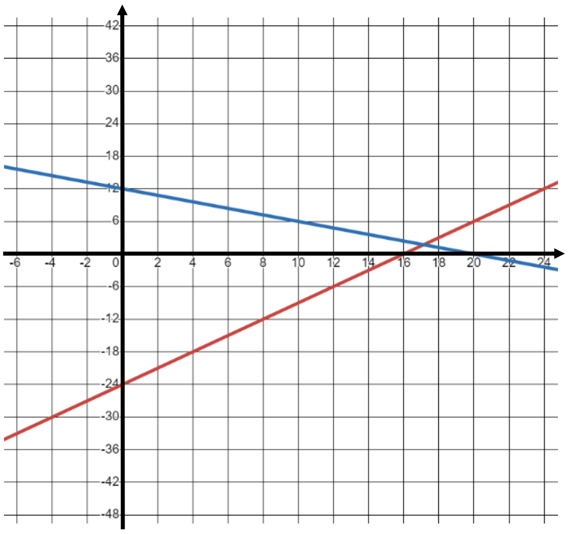
3 **a** $y = \frac{1}{3}x + 2$ **b** $y = -3x$

c $y = \frac{9}{2}x - 4$ **d** $y = -\frac{4}{11}x + 3$

e $y = -\frac{3}{5}x - 1$

4 **a** $y = -\frac{1}{5}x - 4$ **b** $y = \frac{1}{2}x + 8$

5



a $y = -\frac{6}{10}x + 12$

b If the lines were perpendicular, the gradient of Line 2 would be $-\frac{2}{3}$. As $-\frac{6}{10}$ is not equivalent to $-\frac{2}{3}$, the lines are not perpendicular.

Section 7a

Video links:
1) https://www.mathsgenie.co.uk/area-perimeter.html
2) https://www.youtube.com/watch?v=GjS0SrvS0zk&t=4s
3) https://www.mathsgenie.co.uk/surfacearea.php

Area of 2D shapes and surface area of 3D shapes

Key points

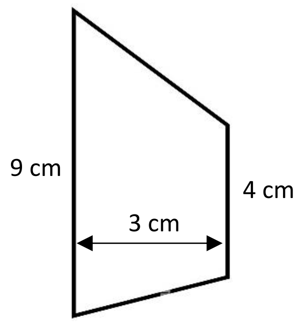
- The area of a 2D shape tells you how much space there is inside it.
- To calculate the area of a 2D shape use the relevant formula:

Shape	Formula	Notes
Rectangle	Area = bh	b is the base and h is the height
Parallelogram	Area = bh	b is the base and h is the perpendicular height
Triangle	Area = $\frac{1}{2}(bh)$	b is the base and h is the perpendicular height
Trapezium	Area = $\frac{1}{2}(a+b)h$	a and b are the parallel sides and h is the perpendicular height
Circle	Area = πr^2	r is the radius

- To find the area of a compound shape, split it into more simple shapes then combine their areas.
- The surface area of a 3D shape is the total area of all of its faces. To calculate it, find the area of each face, then add these together.

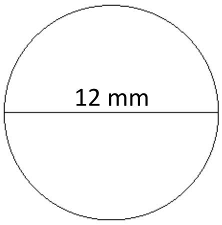
Examples

Example 1 Calculate the area of the trapezium



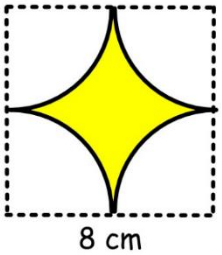
Area = $\frac{1}{2}(a+b)h$ $= \frac{1}{2}(9+4) \times 3$ $= 19.5$ cm^2	**1** The parallel sides are 9 cm and 4 cm, the perpendicular 'height' is 3 cm **2** Units of area are square units

Example 2 Calculate the area of the circle, giving your answer correct to 3 significant figures.



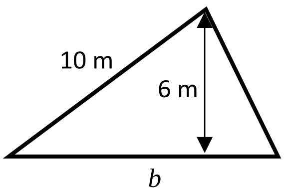
Area = πr^2 $= \pi \times 6^2$ = 113.097 335… = 113 mm^2	**1** The diameter is 12 mm so the radius is 6 mm **2** Round to 3 significant figures

Example 3 Calculate the area of the shaded shape shown.
The dashed lines show a square.
Give your answer correct to 3 significant figures.



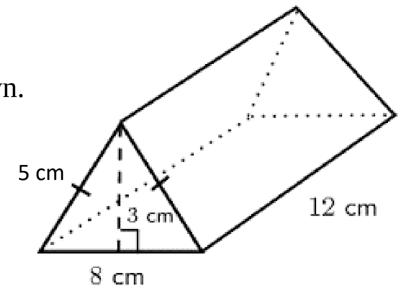
$Area_{\,square} = b^2$ $= 8^2$ = 64 cm^2 $Area_{\,circle} = \pi r^2$ $= \pi \times 4^2$ $= 16\pi$ $Area_{\,shaded} = 64 - 16\pi$ = 13.734 517… = 13.7 cm^2	**1** Start by finding the total area **2** The 4 sectors would combine to make a complete circle **3** Subtract the area of the 4 sectors from the overall square (use exact value for as long as possible) **4** Round to 3 significant figures

Example 4 The scalene triangle shown has an area of 33.6 m^2.
Calculate the length of *b*.



Area = $\frac{1}{2}bh$ $33.6 = \frac{1}{2} \times b \times 6$ $33.6 = 3b$ $b = 11.2$ m	**1** The area is 33.6 m^2 and the height is 6 m **2** Simplify the equation **3** Solve the equation for *b*

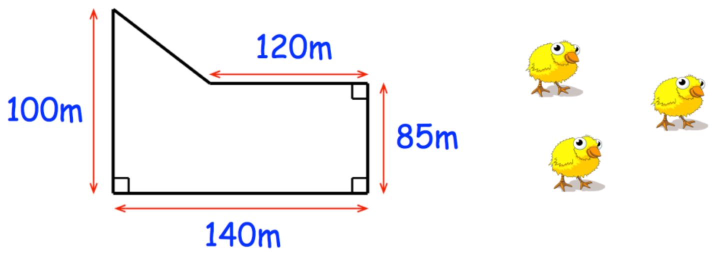
Example 5 Calculate the surface area of the triangular prism shown.



$Area_{front} = \frac{1}{2}bh$ $= \frac{1}{2} \times 8 \times 3$ = 12 cm^2	**1** The front face is a triangle
$Area_{right} = bh$ = 5 × 12 = 60 cm^2	**2** The right-hand face is a rectangle
$Area_{base} = bh$ = 8 × 12 = 96 cm^2	**3** The face on the base is a rectangle
Surface area = 12 × 2 + 60 × 2 + 96	**4** The back face is identical to the front and the left-hand face is identical to the right so do not need to be calculated individually
= 240 cm^2	**5** Units of area are square units

Exercise 7a

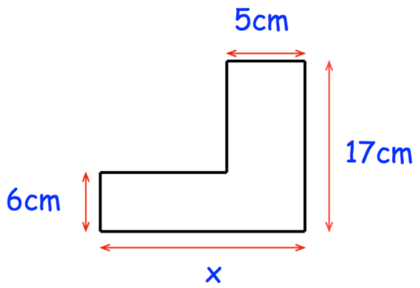
1 Farmer Martin keeps chickens in the field below.



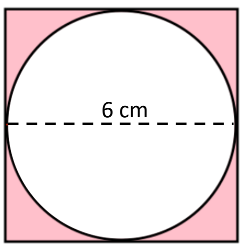
a Calculate the area of the field

b Each chicken needs 3 m^2.
What is the maximum number of chickens Farmer Martin can keep in this field?

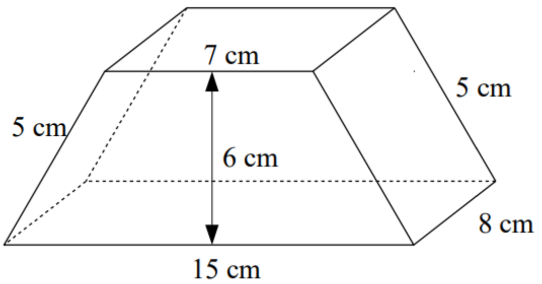
2 The total area of the rectilinear shape shown is 151 cm^2.
Calculate the length of *x*.



3 A circular disk of diameter 6 cm is cut from a square piece of metal of side length 6 cm.
What percentage of the metal is wasted (the wasted metal is shown shaded on the diagram below)?
Give your answer correct to 3 significant figures.



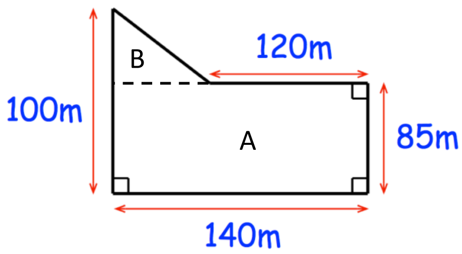
4 The diagram shows a prism.



The cross section of the prism is in the shape of a trapezium.
Calculate the total surface area of the prism.

ANSWERS 7a

1 **a**



$Area_A = 11900 \text{ m}^2$
$Area_B = 150 \text{ m}^2$

Total area = 12050 m^2

b 12050 ÷ 3 = 4016.666…
i.e. Farmer Martin can keep a maximum of 4016 chickens in this field.

2 $x = 16$ cm

3 $Area_{square} = 36 \text{ cm}^2$
$Area_{circle} = 9\pi$
$Area_{wasted} = 36 - 9\pi$

Percentage wasted $= \frac{36 - 9\pi}{36} \times 100 = 21.5\%$ (3 sig fig)

4 $Area_{front} = 66 \text{ cm}^2$
$Area_{top} = 56 \text{ cm}^2$
$Area_{base} = 120 \text{ cm}^2$
$Area_{right} = 40 \text{ cm}^2$

Surface area = 66 + 56 + 120 + 2 × 40 = 322 cm^2

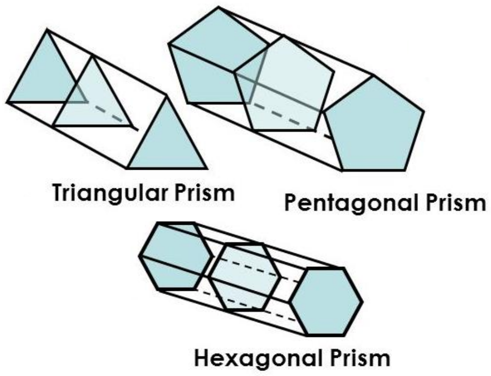
Section 7b

Video link:
https://www.mathsgenie.co.uk/volume.php

Volume of 3D prisms

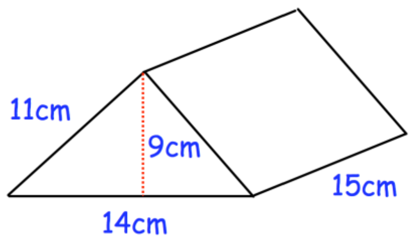
Key points

- The volume of a 3D shape tells you how much space there is inside it.
- A prism is a 3D shape that has a constant cross-section.
- To calculate the volume of a prism:
    1. Find the area of its cross-section (CSA)
    2. Multiply the CSA by the prism's length



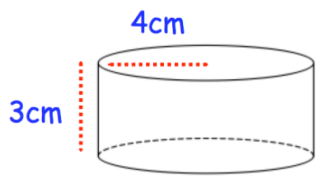
Examples

Example 1 Calculate the volume of the triangular prism



Volume = CSA × length $= \frac{1}{2}(14 \times 9) \times 15$ = 945 cm^3	**1** The cross-section is a triangle **2** Units of volume are cubed units

Example 2 Calculate the volume of the cylinder.
Give your answer correct to 3 significant figures.

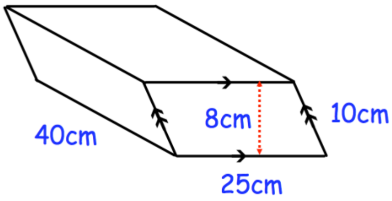


Volume = CSA × length $= \pi \times 4^2 \times 3$ = 150.796 447… = 151 cm^3	**1** The cross-section is a circle **2** Round to 3 significant figures **3** Units of volume are cubed units

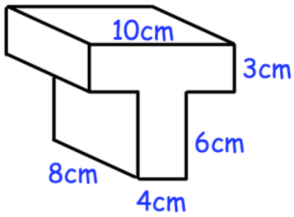
Exercise 7b

1 Calculate the volume of each prism

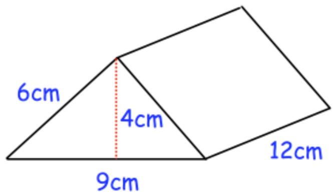
a

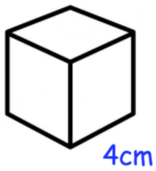


b

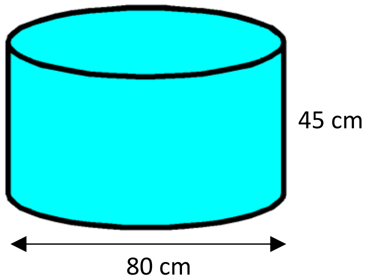


2 The solid triangular prism shown below is made from metal.
The prism is melted down and the metal is used to make a number of cubes of side length 4 cm.
What is the maximum number of cubes that can be made?

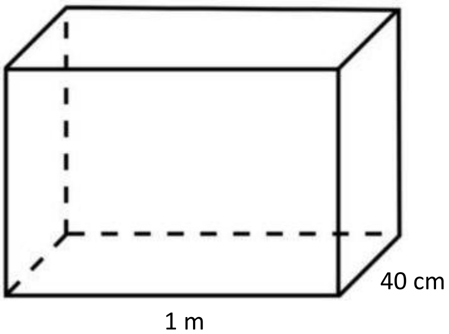




3 The cylindrical vase shown below is full to the brim with water.



The water is poured from the vase into the empty cuboid-shaped water tank shown below.
None of the water is spilled during this process.



How deep will the water be in the water tank?
Give your answer correct to 3 significant figures.

ANSWERS 7b

1 **a** Volume = 8000 cm^3 **b** Volume = 432 cm^3

2 $Volume_{triangular\ prism}$ = 216 cm^3
$Volume_{cube}$ = 64 cm^3

216 ÷ 64 = 3.375
i.e. a maximum of 3 cubes can be made

3 $Volume_{water}$ = 72000π
Cross-section of tank = 100 × 40 = 4000 cm^2

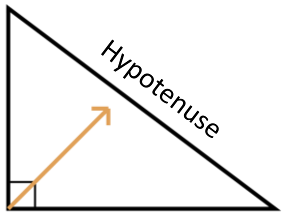
72000π ÷ 4000 = 56.548 667…
i.e. water depth = 56.5 cm (3 sig fig)

Section 8a

Video link:
https://corbettmaths.com/2012/08/19/pythagoras-video/

Pythagoras' Theorem

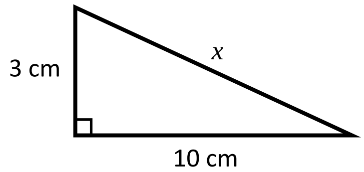
Key points

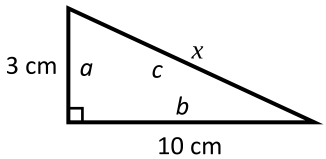


- In a right-angled triangle the side opposite the right angle is called the hypotenuse. This is always the longest side.
- Pythagoras' Theorem states: $a^2 + b^2 = c^2$
  Where c is the length of the hypotenuse and a and b are the lengths of the other two sides.

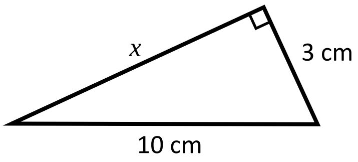
Examples

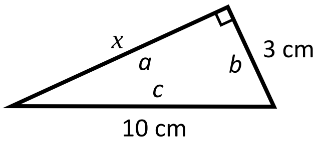
Example 1 Calculate the length of side *x*.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the sides, ensuring the hypotenuse is *c*.
$a^2 + b^2 = c^2$	**2** Substitute the lengths into Pythagoras' Theorem.
$3^2 + 10^2 = x^2$ $9 + 100 = x^2$ $109 = x^2$ $x = \sqrt{109}$ = 10.440 306…	**3** Solve for *x*.
= 10.4 cm	**4** Round your answer to 3 significant figures and write the units in your answer.

Example 2 Calculate the length of side x.
Give your answer correct to 3 significant figures.

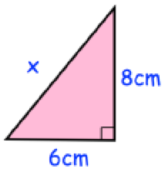


	**1** Always start by labelling the sides, ensuring the hypotenuse is c.
$a^2 + b^2 = c^2$	**2** Substitute the lengths into Pythagoras' Theorem.
$x^2 + 3^2 = 10^2$ $x^2 + 9 = 100$ $x^2 = 100 - 9$ $x^2 = 91$ $x = \sqrt{91}$	**3** Solve for x.
$= 9.539\ 392\ldots$ $= 9.54$ cm	**4** Round your answer to 3 significant figures and write the units in your answer.

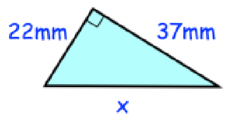
Exercise 8a

1 Calculate the value of x, giving your answers correct to 3 significant figures where appropriate

a

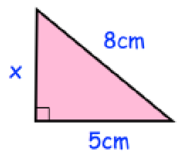


b

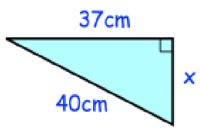


2 Calculate the value of x, giving your answers correct to 3 significant figures where appropriate

a



b



3 From Alton airfield, a plane flies due North for 52 miles, then due East for 186 miles and lands at Brightwater airbase.

Calculate the direct distance from Alton airfield to Brightwater airbase, giving your answer correct to 3 significant figures.

ANSWERS 8a

1 **a** x = 10 cm **b** x = 43.0 mm

2 **a** x = 6.24 cm **b** x = 15.2 cm

3 Distance = 193 miles

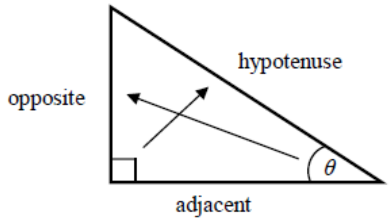
Section 8b

Video links:
1) https://corbettmaths.com/2013/03/30/trigonometry-missing-sides/
2) https://corbettmaths.com/2013/03/30/trigonometry-missing-angles/

Trigonometry in right-angled triangles

Key points

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle θ is called the opposite
  - the side next to the angle θ is called the adjacent.

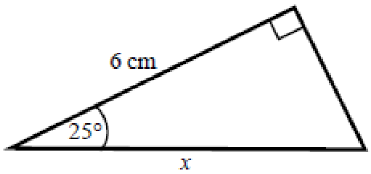


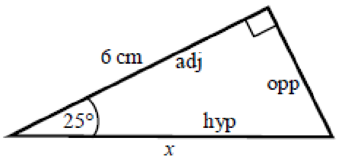
- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle θ, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle θ, $\tan\theta = \frac{\text{opp}}{\text{adj}}$

- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

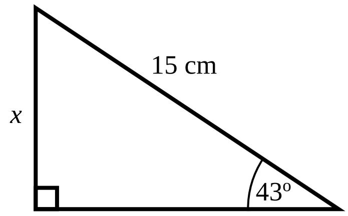
Examples

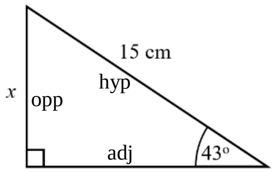
Example 1 Calculate the length of side x.
Give your answer correct to 3 significant figures.



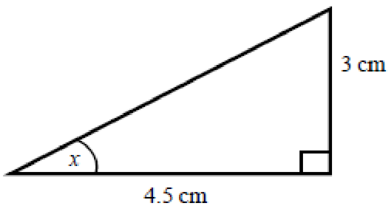
	**1** Always start by labelling the sides.
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	**2** You are given the adjacent and the hypotenuse so use the cosine ratio.
$\cos 25° = \frac{6}{x}$	**3** Substitute the sides and angle into the cosine ratio.
$x = \frac{6}{\cos 25°}$	**4** Rearrange to make x the subject.
$x = 6.620\,267\,5...$	**5** Use your calculator to work out $6 \div \cos 25°$.
$x = 6.62$ cm	**6** Round your answer to 3 significant figures and write the units in your answer.

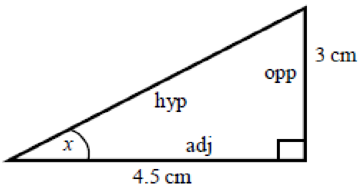
Example 2 Calculate the length of side x.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the sides.
$\sin\theta = \frac{opp}{hyp}$	**2** You are given the opposite and the hypotenuse so use the sine ratio.
$\sin 43° = \frac{x}{15}$	**3** Substitute the sides and angle into the sine ratio.
$x = 15 \times \sin 43°$ $x = 10.229\,975\,4\ldots$	**4** Rearrange to make x the subject and calculate.
$x = 10.2$ cm	**5** Round your answer to 3 significant figures and write the units in your answer.

Example 3 Calculate the size of angle x.
Give your answer correct to 3 significant figures.

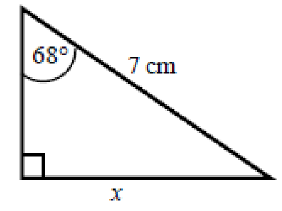


	**1** Always start by labelling the sides.
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \frac{3}{4.5}$	**3** Substitute the sides and angle into the tangent ratio.
$x = \tan^{-1}\left(\frac{3}{4.5}\right)$	**4** Use $\tan^{-1}$ to find the angle.
$x = 33.690\,067\,5...$	**5** Use your calculator to work out $\tan^{-1}(3 \div 4.5)$.
$x = 33.7°$	**6** Round your answer to 3 significant figures and write the units in your answer.

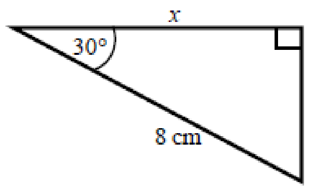
Exercise 8b

1 Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

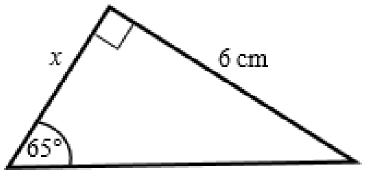
a



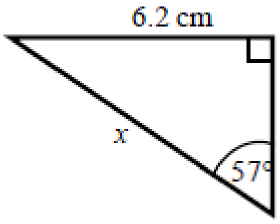
b



c

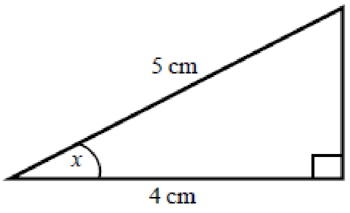


d

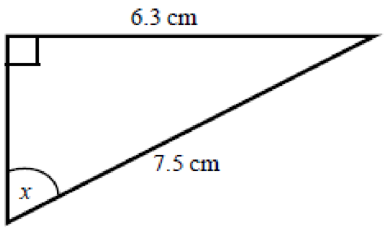


2 Calculate the size of angle *x* in each triangle.
Give your answers correct to 3 significant figures.

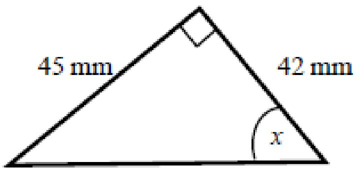
a



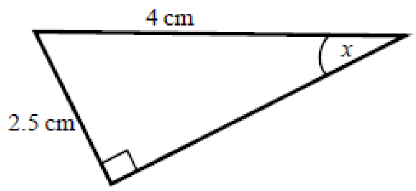
b



c



d



3 The foot of a ladder is on horizontal ground and the top of the ladder is leant against a vertical wall.
The ladder reaches 4.3 metres up the wall and makes an angle of 36° with the wall.
Calculate the length of the ladder, giving your answer correct to 3 significant figures.

ANSWERS 8b

1 **a** $x = 6.49$ cm **b** $x = 6.93$ cm
c $x = 2.80$ cm **d** $x = 7.39$ cm

2 **a** $x = 36.9^{o}$ **b** $x = 57.1^{o}$
c $x = 47.0^{o}$ **d** $x = 38.7^{o}$

3 5.32 metres

Section 9

Video link:
https://www.mathsgenie.co.uk/averages.php

Averages and range

Key points

- Averages are a measure of 'central tendency'. There are three types of average:
  - Mean - this would be the value if the total were shared out equally; find it by adding the data values then dividing this by the number of data values
  - Median - this is the value in the middle of an ordered list (if there are an even number of data values in the list, the median is half-way between the two middle data values)
  - Mode - this is the most common data value. A data set that has a single mode is called 'unimodal', a data set with two modes is called 'bimodal', a data set with three modes is called 'trimodal' and a data set with more than three modes is called 'multimodal'.
- The range is a measure of 'spread'; it is the difference between the largest and smallest data value. To calculate the range, use $Range = largest - smallest$

Examples

Example 1 Calculate the mean of the following list of data:

7, 3, 8, 0, 7, 4, -1, 5, 2

Give your answer correct to 3 significant figures.

$Mean = \frac{7+3+8+0+7+4+(-1)+5+2}{9}$ $= \frac{35}{9}$ = 3.888 888… = 3.89	**1** Add the data values then divide by the number of values (9) **2** Round your answer to 3 significant figures

Example 2 Calculate the median of the following list of data:

7, 3, 8, 0, 7, 4, -1, 5, 2

-1, 0, 2, 3, 4, 5, 7, 7, 8 ↑ (4) Median = 4	**1** The data must be arranged in order **2** The median is the value in the middle of the list

Example 3 Calculate the median of the following list of data:

7, 3, 8, 0, 7, 4, -1, 5, 2, 15

-1, 0, 2, 3, 4, 5, 7, 7, 8, 15 ↑ Median = 4.5	**1** The data must be arranged in order **2** The median is the value in the middle of the list; because there is an even number of data values, this is half-way between the two middle data values

Example 4 Calculate the mode of the following list of data:

7, 3, 8, 0, 7, 4, -1, 5, 2, 15

-1, 0, 2, 3, 4, 5, 7, 7, 8, 15 Mode = 7	**1** The mode is easiest to identify when the data is arranged in order **2** The mode is the data value that occurs most often

Example 5 Calculate the mode of the following list of data:

7, 3, 8, 0, 7, 4, -1, 5, 2, 15

-1, 0, 2, 3, 4, 5, 7, 7, 8, 15 Range = 15 – (-1) = 16	**1** The range is easiest to identify when the data is arranged in order **2** The range is the difference between the largest and smallest data values

Exercise 9

1 Find the mean, median, mode and range for the following data sets, giving your answers to 3 significant figures where necessary

a 9, 1, 3, 6, 7, 8, 9 **b** - 4, 5, - 7, - 1, 2, 9, 2, 5

2 Aarman is conducting an experiment to investigate whether plants grow taller, on average, when fed with Fertilizer A or Fertilizer B. He has grown 10 plants using each fertilizer and has measured their heights; his data is shown below.
Use Aarman's data to write the conclusion for his experiment.

Plant height (cm) Fertilizer A	**Plant height (cm) Fertilizer B**
30.5	31.6
27.9	33.4
30.1	29.2
31.7	32.0
34.8	31.9
28.6	25.4
25.9	32.8
32.4	32.2
30.7	33.1
31.3	32.9

Conclusion:
According to the data above, on average, plants grow taller when they are fed with Fertilizer A / B.
(delete as appropriate)

ANSWERS 9

1 **a**

Mean = 6.14

Median = 7

Mode = 9

Range = 8

b

Mean = 1.38

Median = 2

Mode = 2 and 5 (bimodal)

Range = 16

2 Fertilizer A:

Mean = 30.4 cm

Median = 30.6 cm

Mode = no mode

Fertilizer B:

Mean = 31.5 cm

Median = 32.1 cm

Mode = no mode

Conclusion:
According to the data above, on average, plants grow taller when they are fed with Fertilizer B.

Section 10

Video links:
1) https://corbettmaths.com/2013/06/15/probability/
2) https://www.mathsgenie.co.uk/probability2.php

Probability

Key points

- Probability is a measure of how likely an event is to happen.
- It is measured on a scale from 0 to 1, where 0 represents an impossible event and 1 represents an event that is certain.
- If all possible outcomes in an event are equally likely, then the probability of any of them happening can be calculated using $\frac{number\ of\ successful\ outcomes}{total\ number\ of\ outcomes}$
- The sum of the probabilities of all possible outcomes in an event is always 1.

Examples

Example 1 A bag contains 5 red sweets, 8 yellow sweets and 12 green sweets.
One sweet is to be chosen from the bag at random.
Calculate the probability that the sweet chosen is yellow.

Number of 'successful' outcomes is 8 (8 yellow sweets) Total number of sweets/outcomes = 5 + 8 + 12 = 25 Probability = $\frac{8}{25}$	**1** Identify the number of 'successful' outcomes **2** Identify the total number of outcomes **3** $Probability = \frac{number\ of\ successful\ outcomes}{total\ number\ of\ outcomes}$ Where selecting a yellow sweet is a 'success'

Example 2 A box contains the following number cards.

1	2	2	3	5	6	7	8	8	10

One card is to be selected from the box at random.
Calculate the probability that the card selected is either an even number or a multiple of 5.

Even cards: 2, 2, 6, 8, 8, 10 Multiples of 5: 5, 10	**1** Identify the number of 'successful' outcomes, taking care not to double-count any cards (i.e. 10)
Number of 'successful' outcomes is 7 (7 cards have either even number or multiple of 5)	
Total number of cards/outcomes is 10	**2** Identify the total number of outcomes
Probability = $\frac{7}{10}$	**3** $Probability = \frac{number\ of\ successful\ outcomes}{total\ number\ of\ outcomes}$

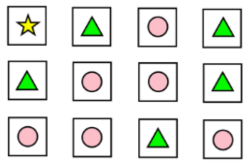
Example 3 The table below shows the probability of scoring each value on a biased dice.
Calculate the probability the dice will score 5.

Score	1	2	3	4	5	6
Probability	0.1	0.2	0.15	0.3	x	0.05

$0.1 + 0.2 + 0.15 + 0.3 + x + 0.05 = 1$	**1** The sum of the probabilities of all possible outcomes in an event is always 1
$0.8 + x = 1$ $x = 0.2$	**2** Solve for x

Exercise 10

1 Kate is going to select one of the shape cards at random.

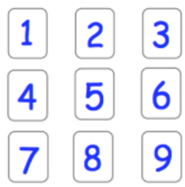


a What is the probability she selects a card with a triangle on it?

b What is the probability she selects a card that does **not** have a circle on it?

c What is the probability she selects a card with either a triangle or a circle on it?

2 Raymond is going to select one of the number cards at random.



a What is the probability he selects a card with a value less than 3?

b What is the probability he selects a card with a square number?

c What is the probability he selects a card with either a square number **or** an even number?

3 There are only pink, yellow, green and blue counters in a bag.
The table below shows the probability that a counter taken at random from the bag will be pink, green or blue.

Colour	Pink	Yellow	Green	Blue
Probability	0.5		0.1	0.2

a Work out the probability that the counter taken is yellow.

b There are 40 counters in the bag in total.
Work out the number of blue counters.

4 The two-way table below gives information about 90 people who sat their driving test last week.

	Under 20 driving lessons	20 or over driving lessons	total
Pass		21	30
Fail	45		
total			90

a Complete the two-way table.

b One of the people is selected at random.
Work out the probability this person passed their driving test.

c One of the people who passed their driving test is selected at random.
Work out the probability this person had under 20 driving lessons.

ANSWERS 10

1 **a** $\frac{5}{12}$ **b** $\frac{6}{12}$ *or* $\frac{1}{2}$ **c** $\frac{11}{12}$

2 **a** $\frac{2}{9}$ **b** $\frac{3}{9}$ *or* $\frac{1}{3}$ **c** $\frac{6}{9}$ *or* $\frac{2}{3}$

3 **a** 0.2 **b** 8 blue counters

4 **a**

	Under 20 driving lessons	20 or over driving lessons	total
Pass	9	21	30
Fail	45	15	60
total	54	36	90

b $\frac{30}{90}$ *or* $\frac{1}{3}$

c $\frac{9}{30}$ *or* $\frac{3}{10}$