Anglo European Sixth Form

Summer Transition Work

Subject: Mathematics

Exam Board: IB

Qualification: IB SL Mathematics: Applications and Interpretation

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Compulsory tasks:

Complete **all questions from Section A** of the 'IB SL Mathematics: Applications & Interpretation Transition Work' booklet and check your answers. If you score less than 75% in a topic, work through the examples and questions linked to that topic in Section B of the booklet to help improve your understanding ready for September. If you need further support with the topic, use the additional recommended resources for more practice.

Advisory tasks:

Complete the 'IB SL Mathematics: Applications and Interpretation Data Task' and check your answers.

ANGLO EUROPEAN SCHOOL

IB SL Mathematics: Applications & Interpretation Transition Work

Examples, Practice Questions & Answers:

10 Key Topics to prepare you for IB Maths:

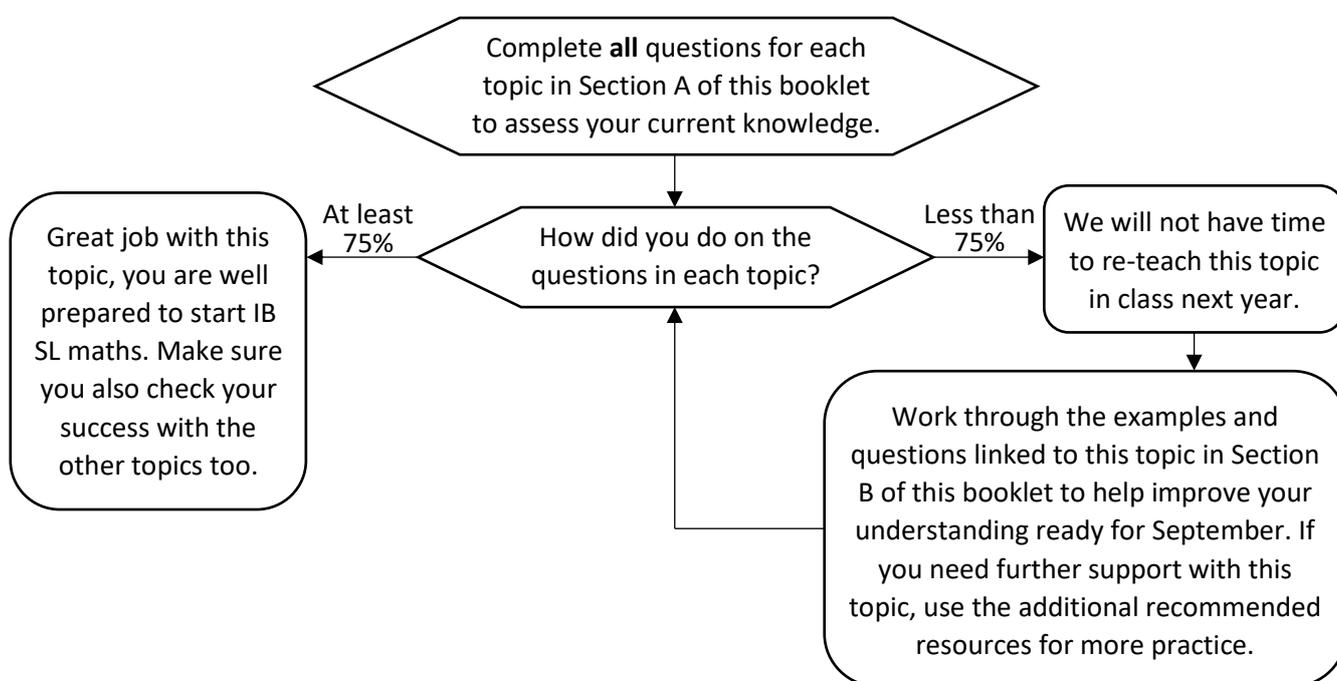|	Topic	☺	😐	☹
1.	Calculating and substituting with negative numbers			
2.	Percentages			
3.	Rounding and error intervals			
4.	Rearranging equations			
5.	Solving linear equations and inequalities			
6.	Linear Graphs			
7.	Area of 2D shapes & surface area and volume of 3D prisms			
8.	Pythagoras and trigonometry			
9.	Averages and range			
10.	Probability			

Name: _____

Introduction

Congratulations on choosing to study IB SL Mathematics: Applications and Interpretation. This booklet will help you prepare by brushing up on some of the skills you have learned at GCSE. We will not have time to re-teach these topics in class, but you will need to be **fluent** in them to be able to learn the IB content, so if you don't currently have a good grasp of these topics you need to work on them **NOW** so that you can start with confidence.

Mathematical fluency does not simply mean that you have met this topic before and think that you remember how to do it. To reach fluency, you must be able to **quickly** and **accurately** recall concepts and methods.

There will be a test on the topics from this booklet in the first week of term. It is expected that IB SL students will demonstrate an <u>solid</u> understanding of these topics.

How to use this booklet:

Complete **all** questions for each topic in Section A of this booklet to assess your current knowledge.

How did you do on the questions in each topic?

At least 75%

Great job with this topic, you are well prepared to start IB SL maths. Make sure you also check your success with the other topics too.

Less than 75%

We will not have time to re-teach this topic in class next year.

Work through the examples and questions linked to this topic in Section B of this booklet to help improve your understanding ready for September. If you need further support with this topic, use the additional recommended resources for more practice.

Please bring your completed and self-marked work from this booklet to your first maths lesson in September to show your teacher.

Differences between GCSE and IB SL maths:

GCSE Maths	IB SL Maths
You have an exercise book to keep all your work together.	You will need to keep neat, accurate and well-ordered notes and work in your folder.
It's the answer that matters most, but you should show working.	Your course is about applying maths to real-life situations and interpreting the findings. Showing your method is vital to be able to justify your findings. This is great preparation for solving problems in a place of work.
You need to know how to solve problems without using a calculator.	A big part of your course is learning how to use technology (including your graphical calculator) to solve problems and you need your calculator for all of your exams. However, it is extremely beneficial to maintain your basic mental arithmetic skills to allow you to solve problems efficiently as well as to check whether the answer from your calculator is sensible.
How you present your work is not overly important, as long as you get there in the end.	How you present your work can make a big difference to whether anyone can understand your method and therefore have confidence in your findings. If you use technology (e.g. graphical calculator) to find the answer, you must clearly show how you used this.
Your final grade comes only from the final exams.	You will sit 2 final exams at the end of Upper Sixth, but this is only 80% of your final grade. The remaining 20% comes from your Internal Assessment (IA), i.e. coursework. In this piece of work, you will apply the maths you have learned to a project with real-life data; you will need to show and justify your methods in order to reach a convincing conclusion.

Additional website resources:

- Dr Frost – Click 'Practise' and choose 'Practise by Topic' (requires free sign-up if you don't yet have an account) https://www.drfrostmaths.com

- Maths Genie – GCSE questions with model solutions; also has videos. https://www.mathsgenie.co.uk/gcse.html

- Corbett Maths – GCSE questions with model solutions; also has videos. https://corbettmaths.com/contents/

- Exam solutions – videos and GCSE questions with mark schemes https://www.examsolutions.net/gcse-maths/

- Khan academy – linked to American school syllabus, but has very clear videos for many topics https://www.khanacademy.org/math/algebra-home

- Mathswatch – interactive GCSE questions and videos. If you studied with us for Year 11 you will already have an account; click 'Videos' and search for the topic you wish to study then click 'Interactive questions' https://vle.mathswatch.co.uk/vle/

Section A: Assess your current knowledge

Complete all questions in this section (pg 5 – 12) and use the answers provided (pg 13 – 17) to mark your work. This section should take you approximately 2 hours to complete.

If you score less than 6 out of 8 in any topic, work through the examples and questions linked to that topic in Section B of this booklet to help improve your understanding ready for September.

Topic 1: Calculating and substituting with negative numbers

1) Work out:
   a) 27 + (-13)

   b) (-48) – (-16)

   c) (-20) ÷ 4

   d) $(-3)^2$

2) If $x = 5$, $y = -2$ and $z = -3$, work out:
   a) $z + y$

   b) xy

   c) $2x - yz$

   d) $\dfrac{y^3}{x - z}$

Topic 2: Percentages

1) James scored 18 out of 24 on his recent science test. What percentage did James score?

2) Find:
   a) 20% of 36

   b) 45% of 80

   c) 17.5% of 236

3) a) Increase 58 by 10%

   b) Decrease 620 by 32%

4) When a ball is dropped it bounces and then rises to 80% of the height it was dropped from. The ball continues to bounce, each time rising to 80% of the height of its previous rise.

   The ball is dropped from a height of 4 metres.

   a) Calculate the height the ball will rise to after its second bounce.

   b) After how many bounces will the ball first fail to rise to a height of 1 metre?

Topic 3: Rounding and error intervals

1) Round:
   a) 52631.8 to the nearest hundred

   b) 381.8249 to 2 decimal places

   c) 3045.1298 to 3 significant figures

   d) 0.0017962 to 3 significant figures

2) Complete the error interval for:
   a) the length of a lizard, L, is 16 cm, correct to the nearest centimetre

     ……… ≤ L < ………

   b) a number, N, is rounded to 1 decimal place and the result is 3.2

     ……… ≤ N < ………

Topic 4: Rearranging equations

Make y the subject of each equation:
   a) $x = 5 + y$

   b) $x = 12 - y$

   c) $x = 3y$

   d) $x = \dfrac{y}{5}$

   e) $x = 2y + 3$

   f) $x = \dfrac{7}{y+z}$

   g) $x = 16y^2$

   h) $x = \dfrac{4}{3}\pi y^3$

Topic 5: Solving linear equations and inequalities

1) Solve:

a) $6x + 5 = 26$

b) $\frac{x}{3} - 6 = 0$

c) $\frac{3(x + 7)}{4} = 6$

d) $4x - 12 = 7x + 3$

2) Solve:

a) $x + 11 > 16$

b) $5x - 2 \leq 19$

c) $20 \geq \frac{x + 4}{2}$

d) $19 - 3x < 7$

Topic 6: Linear graphs

1) Write down the gradient and y-intercept of the graph with the following equation:

  a) $y = 2x + 1$

  b) $y = 6 - 5x$

  c) $3y + 15 = 2x$

  d) $y = -4$

2) Write down the equation of the line:

a)

b)
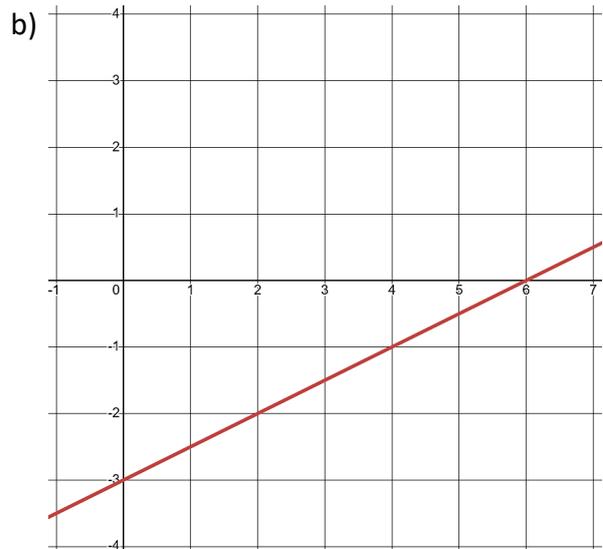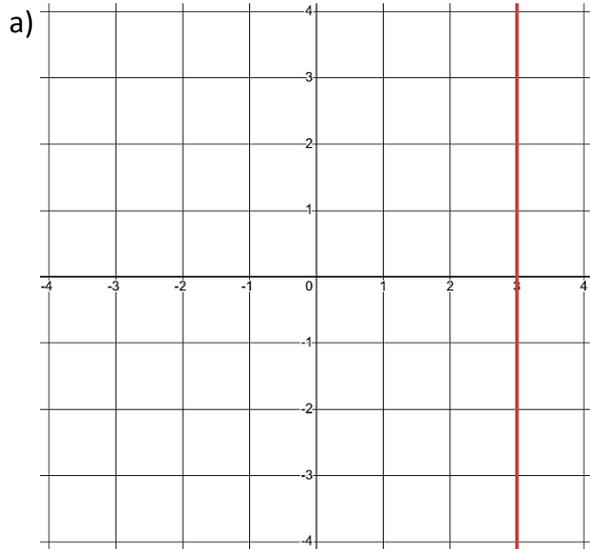
3) Write down the equation of the line that is parallel to Line 1 and passes through point A:

Line 1
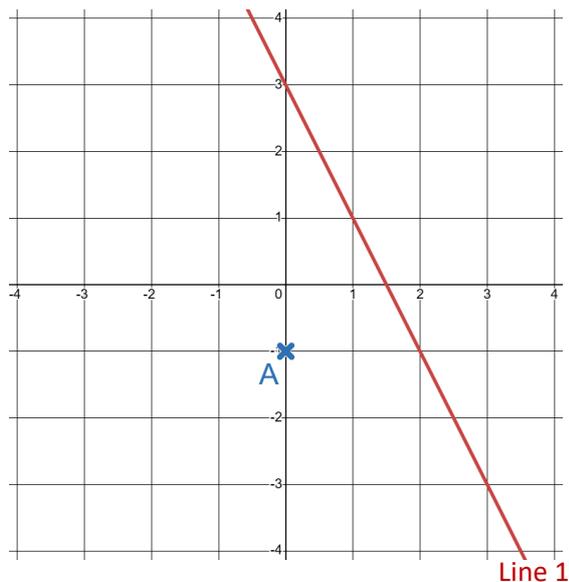
4) Line 2 passes through the points (3, 7) and (-1, 10).
  Write down the equation of the line that is perpendicular to Line 2 and passes through the point (0, 5).

Topic 7: Area of 2D shapes & surface area and volume of 3D prisms

1) Calculate the area of the shaded section.
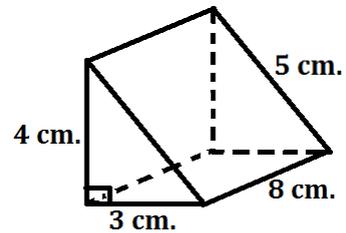   Give your answers either as exact values or rounded to the nearest whole number.

a)

11 cm

5 cm

12 cm

b)

36 m

c)

6 cm

6 cm

10 cm

10 cm
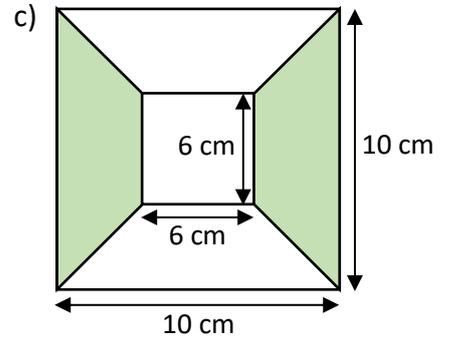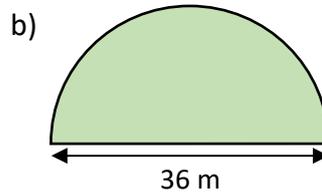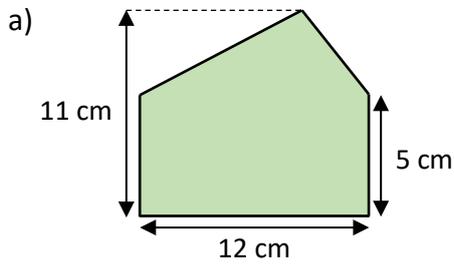
2) The area of the shape below is 635 cm². Calculate the value of x.

25 cm

15 cm

x cm

32 cm
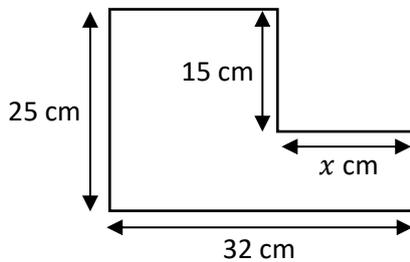
3) a) Calculate the surface area of the prism on the right.

b) Calculate the volume of the prism on the right.
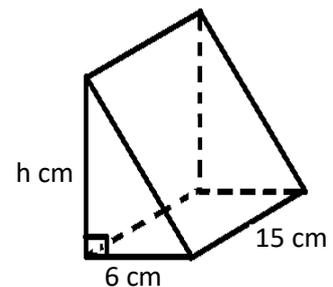
5 cm.

4 cm.

3 cm.

8 cm.

4) Calculate the volume of the cylinder on the right.
   Give your answer correct to 1 decimal place.

7 mm

26 mm
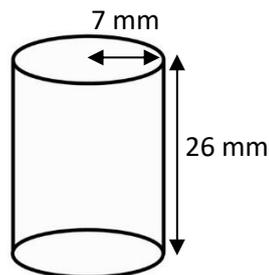
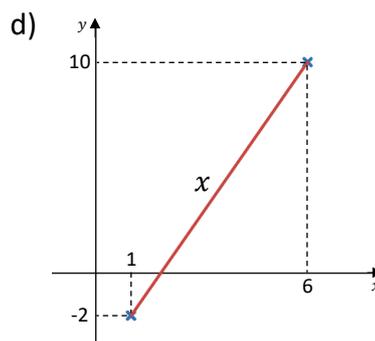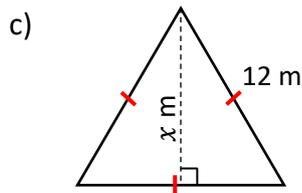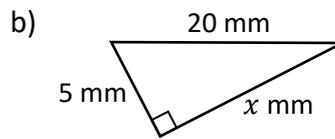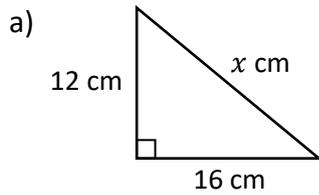5) The volume of the triangular prism on the right is 810 cm³.
   Calculate the value of h.

h cm

6 cm

15 cm

Topic 8: Pythagoras and trigonometry

1) Calculate the length of x.

   Give your answers either as exact values or rounded to 3 significant figures.

   a)

   12 cm

   x cm

   16 cm

   b)

   20 mm

   5 mm

   x mm

   c)

   12 m

   x m

   d)
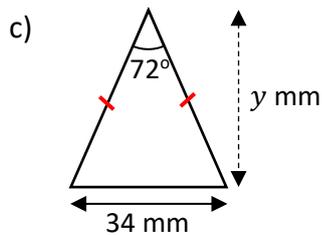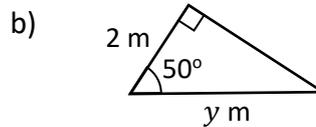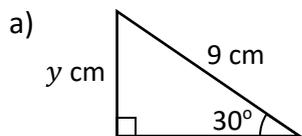
   10

   x

   1

   6

   -2

2) Calculate the length of y.

   Give your answers either as exact values or rounded to 3 significant figures.

   a)

   9 cm

   y cm

   30°

   b)

   2 m

   50°

   y m

   c)

   72°

   y mm

   34 mm

3) Find the size of angle z.

   Give your answer correct to the nearest whole degree.

   12 cm

   10 cm
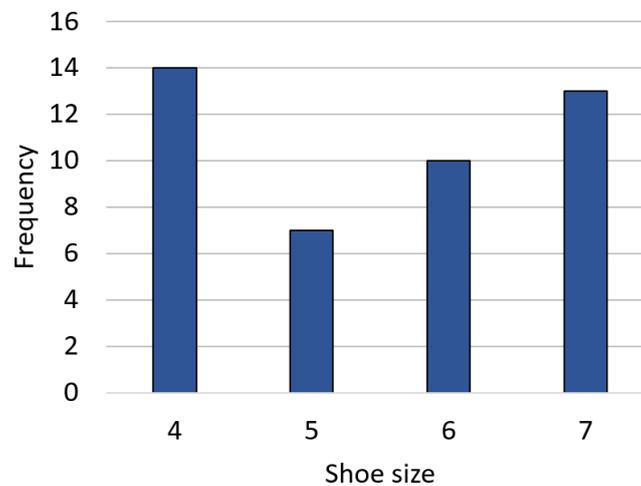
   z°
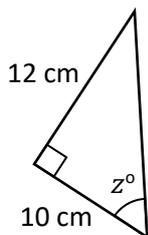
Topic 9: Averages and range

1) Here is a list of numbers:

49, 34, 12, 10, 53, 20, 60, 12, 90, 100, 33

Use the list of numbers to find the:

a) range

b) mode

c) median

d) mean

2) The bar chart below shows the shoe sizes of a group of students:

Use the bar chart to find the:

a) range of the shoe sizes

b) modal shoe size

c) median shoe size

d) mean shoe size

Topic 10: Probability

1) The probability that it will rain tomorrow is 24%.
   Write down the probability that it will not rain tomorrow.

2) Jo has a set of 20, cards numbered from 1 to 20.
   a) One of the cards is selected at random. Find the probability that the selected card is:

   i) a multiple of 3

   ii) both a square number and a factor of 36

   iii) either an even number or a prime number

   b) One card is removed from the set at random and is found to be the number 10.
      It is not replaced.
      Another card is then selected at random from the remaining cards.
      What is the probability that this selected card is an even number?

3) The two-way table below shows information about the students who study Japanese, History and Science. None of the students study more than one of these subjects.

	Japanese	History	Science	Total
Male	11	21	23	55
Female	16	14	19	49
Total	27	35	42	104

   a) One of the students is selected at random. Find the probability that the selected student:

   i) studies History

   ii) is female and studies Japanese

   b) One of the male students is selected at random. Find the probability that he studies Science.

Topic 1: Calculating and substituting with negative numbers
Answers

1) a) 14   (1 mark)   2) a) -5   (1 mark)

   b) -32   (1 mark)   b) -10   (1 mark)

   c) -5   (1 mark)   c) 4   (1 mark)

   d) 9   (1 mark)   d) -1   (1 mark)

If you scored at least 6 out of 8 marks in this topic, you are well prepared to start IB SL Maths, well done! Make sure you also check your success with the other topics too.

If you scored less than 6 out of 8 marks in this topic, work through the examples and questions linked to this topic on **pg 18 – 20** in Section B of this booklet to help improve your understanding ready for September.

Topic 2: Percentages
Answers

1) 75%   (1 mark)   3) a) 63.8   (1 mark)

                                         b) 421.6   (1 mark)

2) a) 7.2   (1 mark)

   b) 36   (1 mark)   4) a) 2.56 m   (1 mark)

   c) 41.3   (1 mark)   b) 7 bounces (1 mark)

If you scored at least 6 out of 8 marks in this topic, you are well prepared to start IB SL Maths, well done! Make sure you also check your success with the other topics too.

If you scored less than 6 out of 8 marks in this topic, work through the examples and questions linked to this topic on **pg 21 – 26** in Section B of this booklet to help improve your understanding ready for September.

Topic 3: Rounding and error intervals
Answers

1) a) 52600   (1 mark)

  b) 381.82   (1 mark)

  c) 3050   (1 mark)

  d) 0.00180   (1 mark)
     Note: <u>must</u> include final 0

2) a) $15.5 \leq L < 16.5$
      (1 mark for 15.5, 1 mark for 16.5)

  b) $3.15 \leq N < 3.25$
      (1 mark for 3.15, 1 mark for 3.25)

If you scored at least 6 out of 8 marks in this topic, you are well prepared to start IB SL Maths, well done! Make sure you also check your success with the other topics too.

If you scored less than 6 out of 8 marks in this topic, work through the examples and questions linked to this topic on **pg 27 – 31** in Section B of this booklet to help improve your understanding ready for September.

Topic 4: Rearranging equations
Answers

a) $y = x - 5$ (1 mark)

b) $y = 12 - x$ (1 mark)

c) $y = \frac{x}{3}$ (1 mark)

d) $y = 5x$ (1 mark)

e) $y = \frac{x-3}{2}$ (1 mark)

f) $y = \frac{7}{x} - z$ (1 mark)

g) $y = \sqrt{\frac{x}{16}}$ or $y = \frac{\sqrt{x}}{4}$ (1 mark)

h) $y = \sqrt[3]{\frac{3x}{4\pi}}$ (1 mark)

If you scored at least 6 out of 8 marks in this topic, you are well prepared to start IB SL Maths, well done! Make sure you also check your success with the other topics too.

If you scored less than 6 out of 8 marks in this topic, work through the examples and questions linked to this topic on **pg 32 – 34** in Section B of this booklet to help improve your understanding ready for September.

Topic 5: Solving linear equations and inequalities
Answers

1) a) $x = 3.5$ (1 mark)

   b) $x = 18$ (1 mark)

   c) $x = 1$ (1 mark)

   d) $x = -5$ (1 mark)

2) a) $x > 5$ or $5 < x$ (1 mark)

   b) $x \leq 4.2$ or $4.2 \geq x$ (1 mark)

   c) $x \leq 36$ or $36 \geq x$ (1 mark)

   d) $x > 4$ or $4 < x$ (1 mark)

If you scored at least 6 out of 8 marks in this topic, you are well prepared to start IB SL Maths, well done! Make sure you also check your success with the other topics too.

If you scored less than 6 out of 8 marks in this topic, work through the examples and questions linked to this topic on **pg 35 – 40** in Section B of this booklet to help improve your understanding ready for September.

Topic 6: Linear graphs
Answers

1) a) Gradient = 2
      y-intercept = 1   (1 mark)

   b) Gradient = -5
      y-intercept = 6   (1 mark)

   c) Gradient = $\frac{2}{3}$
      y-intercept = -5   (1 mark)

   d) Gradient = 0
      y-intercept = -4   (1 mark)

2) a) $x = 3$ (1 mark)

   b) $y = \frac{1}{2}x - 3$ (1 mark)

3) $y = -2x - 1$ (1 mark)

4) $y = \frac{4}{3}x + 5$ (1 mark)

If you scored at least 6 out of 8 marks in this topic, you are well prepared to start IB SL Maths, well done! Make sure you also check your success with the other topics too.

If you scored less than 6 out of 8 marks in this topic, work through the examples and questions linked to this topic on **pg 41 – 50** in Section B of this booklet to help improve your understanding ready for September.

Topic 7: Area of 2D shapes & surface area and volume of 3D prisms
Answers

1) a) 96 cm^2 (1 mark)

   b) 162π cm^2 or   509 cm^2 (1 mark)

   c) 32 cm^2 (1 mark)

2) x = 11 cm   (1 mark)

3) a) 108 cm^2 (1 mark)

   b) 48 cm^3 (1 mark)

4) 4002.4 mm^2 (1 mark)

5) h = 18 cm   (1 mark)

If you scored at least 6 out of 8 marks in this topic, you are well prepared to start IB SL Maths, well done! Make sure you also check your success with the other topics too.

If you scored less than 6 out of 8 marks in this topic, work through the examples and questions linked to this topic on **pg 51 – 58** in Section B of this booklet to help improve your understanding ready for September.

Topic 8: Pythagoras and trigonometry
Answers

1) a) x = 20 cm   (1 mark)

   b) $x = 5\sqrt{15}$ mm   or   19.4 mm   (1 mark)

   c) $x = 6\sqrt{3}$ m   or   10.4 m   (1 mark)

   d) x = 13 units   (1 mark)

2) a) y = 4.5 cm   (1 mark)

   b) y = 3.11 m   (1 mark)

   c) y = 23.4 mm   (1 mark)

3) z = 50º   (1 mark)

If you scored at least 6 out of 8 marks in this topic, you are well prepared to start IB SL Maths, well done! Make sure you also check your success with the other topics too.

If you scored less than 6 out of 8 marks in this topic, work through the examples and questions linked to this topic on **pg 59 – 65** in Section B of this booklet to help improve your understanding ready for September.

Topic 9: Averages and range
Answers

1) a) 90   (1 mark)   2) a) 3   (1 mark)

    b) 12   (1 mark)   b) 4   (1 mark)

    c) 34   (1 mark)   c) 6   (1 mark)

    d) 43   (1 mark)   d) 5.5   (1 mark)

If you scored at least 6 out of 8 marks in this topic, you are well prepared to start IB SL Maths, well done! Make sure you also check your success with the other topics too.

If you scored less than 6 out of 8 marks in this topic, work through the examples and questions linked to this topic on **pg 66 – 69** in Section B of this booklet to help improve your understanding ready for September.

Topic 10: Probability
Answers

1) 76%   (1 mark)   3) a) i) $\frac{35}{104}$ (1 mark)

2) a) i) $\frac{6}{20}$ or $\frac{3}{10}$ or 0.3 or 30%   ii) $\frac{16}{104}$ (1 mark)

               (1 mark)

    ii) $\frac{3}{20}$ or 0.15 or 15%   b) $\frac{23}{55}$ (1 mark)

               (1 mark)

    iii) $\frac{17}{20}$ or 0.85 or 85%

               (1 mark)

If you scored at least 6 out of 8 marks in this topic, you are well prepared to start IB SL Maths, well done! Make sure you also check your success with the other topics too.

If you scored less than 6 out of 8 marks in this topic, work through the examples and questions linked to this topic on **pg 70 – 73** in Section B of this booklet to help improve your understanding ready for September.

Section B: Improve your understanding if necessary

Topic 1

Video links:
1) https://www.mathsgenie.co.uk/negativenumbers.php
2) https://www.mathsgenie.co.uk/substitution.php

Calculating and substituting with negative numbers

Key points

- When a negative number is added to a starting value, the answer is always smaller than the starting value, i.e. adding a negative is equivalent to subtraction.
  Examples:
  a) 8 + (-3) = 8 − 3 = 5
  b) (-2) + (-5) = (-2) − 5 = -7
  c) (-18) + (-7) = (-18) − 7 = -25

- When a negative number is subtracted from a starting value, the answer is always greater than the starting value, i.e. subtracting a negative is equivalent to addition.
  Examples:
  a) 8 − (-3) = 8 + 3 = 11
  b) (-2) − (-5) = (-2) + 5 = 3
  c) (-18) − (-7) = (-18) + 7 = -11

- When a negative number is multiplied/divided by a positive number (or a positive number is multiplied/divided by a negative number), the answer is always negative.
  Examples:
  a) 3 x (-4) = -12
  b) (-5) x 2 = -10
  c) (-12) ÷ 6 = -2
  d) 14 ÷ (-2) = -7

- When two negative numbers are multiplied/divided, the answer is always positive.
  Examples:
  a) (-6) x (-3) = 18
  b) $(-3)^2$ = (-3) x (-3) = 9
  c) (-6) ÷ (-3) = 2

- When substituting into algebraic expressions, remember that terms without an operation between them are always **multiplied**.
  Examples:
  a) If a = -7 and b = 13, evaluate: 5ab

$5ab = 5 \times (-7) \times 13$ $= (-35) \times 13$ $= -455$	**1** 5ab means $5 \times a \times b$ **2** A positive and negative multiplied makes a negative answer.

b) If $x = -5$, $y = -8$ and $z = -3$, evaluate: $x^2 + 2y - 6z$

$x^2 + 2y - 6z = (-5)^2 + 2 \times (-8) - 6 \times (-3)$ $= 25 + (-16) - (-18)$ $= 25 - 16 + 18$ $= 27$	**1** $2y$ means $2 \times y$ and $6z$ means $6 \times z$ **2** Two negative numbers multiply to make a positive and a positive and negative multiplied makes a negative. **3** Adding a negative number is equivalent to subtraction and subtracting a negative number is equivalent to addition.

c) If $f = -2$, $g = -6$ and $h = -10$, evaluate: $fg(g - h \div f)$

$fg(g - h \div f) = (-2) \times (-6) \times \big((-6) - (-10) \div (-2)\big)$ $= (-2) \times (-6) \times \big((-6) - 5\big)$ $= (-2) \times (-6) \times (-11)$ $= \quad 12 \quad \times (-11)$ $= \quad -132$	**1** fg means $f \times g$ and $fg(g...$ means $fg \times (g...$ **2** Remember to complete the calculation in the correct order. **3** Two negative numbers multiply/divide to make a positive and a positive and negative multiplied makes a negative.

Exercise 1

1. When a = -5, b = 3 and c = -2 find the value for the expressions.

a) 5a
b) 5a + 10c
c) 2(a + c)
d) c(5 + a)
e) 9(b – c)
f) ab – c
g) a^2 + b
h) -2(a^2 + b)

2. Use the formula y = 4f – 7 to complete the questions.
a) Find the value of y when f = -2

b) Find the value of f when y = -17

3. Use the formula X = -3(a^2 + b) to complete the questions.
a) Find the value of X when a = -4 and b = 8
b) Find the value of a when X = -60, and b = -5

1. When a = -5, b = 3 and c = -2 find the value for the expressions.

a) 5a   -25

b) 5a + 10c   -45

c) 2(a + c)   -14

d) c(5 + a)   0

e) 9(b – c)   45

f) ab – c   -13

g) $a^2 + b$ 28

h) $-2(a^2 + b)$ -56

2. Use the formula y = 4f – 7 to complete the questions.

a) Find the value of y when f = -2

y = -15

b) Find the value of r when y = -17

f = -2.5

3. Use the formula $X = -3(a^2 + b)$ to complete the questions.

a) Find the value of X when a = -4 and b = 8   X = -72

b) Find the value of a when X = -60, and b = -5   a = 5 or -5

Writing percentages and finding percentages

Key points

- 'Per cent' means 'out of 100', so a percentage is always a measure out of 100. This means it is easy to write a percentage as a fraction, e.g. 59% = $\frac{59}{100}$

- To write a fraction as a percentage, if possible, first create an equivalent fraction with a denominator of 100, e.g. 13 out of 20 = $\frac{13}{20}$ = $\frac{65}{100}$ = 65%.

- If it is not possible to create an equivalent fraction with a denominator of 100, then first convert the fraction to a decimal by dividing the numerator by the denominator, then multiply this by 100 to make the percentage, e.g. 5 out of 8 = 5 ÷ 8 = 0.625 = 62.5%

  x 100

- To find a percentage of an amount you can use a number of different methods:
    1. Find the equivalent fraction of the amount

       e.g. find 40% of 230

       $$40\% = \frac{40}{100} = \frac{2}{5}$$

       So, find $\frac{2}{5}$ of 230

       230 ÷ 5 x 2 = 92

       i.e. 40% of 230 = 92

    2. Multiply the amount by the equivalent decimal

       e.g. find 32% of 45

       32% = 0.32

       So, calculate 45 x 0.32

       = 14.4

       i.e. 32% of 45 = 14.4

    3. Use a combination of percentages to 'build' the required percentage

       e.g. find 76% of 860

       76% = 25% x 3 + 1%

       25% of 860 = $\frac{1}{4}$ of 860 = 860 ÷ 4 = 215

       1% of 860 = $\frac{1}{100}$ of 860 = 860 ÷ 100 = 8.6

       So, 76% of 860 = 215 x 3 + 8.6

       = 653.6

1) Using any method you prefer, work out the following.
   Give your answer to 2 d.p. where appropriate.

a) 72 as a percentage of 200

b) 24 as a percentage of 50

c) 19 as a percentage of 24

d) 7 as a percentage of 8

2) Here is a table of sales from a clothes store in a week.

The manager wants to find out the percentage of returns. Which of the items has the highest percentage of returns compared to the sales of that item?

Type	Number of sales	Number of returns
Dresses	23	12
Trousers	11	6
Hats	6	3
T-shirts	64	33

3) Hannah is paid £280.
   She spends 30% on her rent, 25% on food and bills and saves the rest.

   (a) How much does Hannah spend on rent?
   (b) How much does Hannah spend on food and bills?
   (c) How much does Hannah save?

4) Maria has invested different amounts of money in 3 different companies. Over the past year, her investments have increased in value by the percentages shown in the table below.

   Which investment has earned Maria the most money over the past year?

Company	Percentage gain	Amount invested
Pineapple	2.5%	£240
Tech smart	8.1%	£60
Coffee world	4.2%	£150

1) a) $\frac{72}{200}$ = 36%

   b) $\frac{24}{50}$ = 48%

   c) $\frac{19}{24}$ = 79.1666…%

   d) $\frac{7}{8}$ = 87.5%

2) Dresses = $\frac{12}{23}$ = 52.173…%

   Trousers = $\frac{6}{11}$ = 54.545…%

   Hats = $\frac{3}{6}$ = 50%

   T-shirts = $\frac{33}{64}$ = 51.5625%

   Trousers have the highest percentage of returns compared to sales.

3) a) 30% of £280 = £84

   b) 25% of £280 = £70

   c) £280 − £84 − £70 = £126

4) Pineapple = 2.5% of £240 = £6

   Tech smart = 8.1% of £60 = £4.86

   Coffee world = 4.2% of £150 = £6.30

   Coffee world has made Maria the most money over the past year.

Percentage increase/decrease

Key points

- To increase/decrease an amount by a percentage you can use two different methods:
    1. Find the required percentage of the amount then add/subtract this from the original amount

       e.g.1) increase 80 by 23%

       23% of 80 = 18.4

       80 + 18.4 = 98.4

       i.e. increase 80 by 23% = 98.4

       e.g.2) decrease 80 by 23%

       23% of 80 = 18.4

       80 − 18.4 = 61.6

       i.e. decrease 80 by 23% = 61.6

    2. Work out what the final percentage will be then find this percentage of the amount (remember that the original amount is always worth 100%)

       e.g.1) increase 72 by 35%

       100% + 35% = 135%

       135% of 72 = 97.2

       i.e. increase 72 by 35% = 97.2

       e.g.2) decrease 72 by 35%

       100% − 35% = 65%

       65% of 72 = 46.8

       i.e. decrease 72 by 35% = 46.8

Exercise 2b

1) Calculate the following:

   a) Increase 4584 by 24%

   b) Decrease 3815 by 65%

   c) Increase 763 by 2.5%

   d) Decrease 850 by 0.48%

2) A mathematician has her salary of £2000 per month increased by 6%.

60% of her new salary pays for bills, rent, etc.

$\frac{1}{4}$ of her new salary is spent on food.

She donates 2% of her new salary to charity.

How much does she have left per month for savings?

3) Zara wants to buy 72 candles.
Each candle costs £4.80

   There is a special offer

   Work out the cost of buying 72 candles
   using the special offer.

Special Offer
Candles   £4.80 each
Buy 60 or more candles and get 15% off the total cost.

4) An empty flowerpot has a mass of 800g.
The mass of the flowerpot increases to 4kg when filled with soil.

   A different flowerpot is 25% lighter but holds 40% more soil.
   Calculate the mass of this flowerpot when it is full of soil.

ANSWERS 2b

1) a) 4584 x 1.24 = 5684.16

b) 3815 x 0.35 = 1335.25

c) 763 x 1.025 = 782.075

d) 850 x 0.9952 = 845.92

2) Increase £2000 by 6% = 2000 x 1.06 = £2120

60% of £2120 = £1272

$\frac{1}{4}$ of £2120 = £530

2% of £2120 = £42.40

£2120 – £1272 – £530 – £42.40 = £275.60

£275.60 is left for the mathematician to save

3) Full price: 72 candles = 72 x £4.80 = £345.60

Decrease £345.60 by 15% = £293.76

Using the special offer, 72 candles costs £293.76

4) Mass of soil = 4 kg – 800 g = 3.2 kg

Mass of 2nd flowerpot: decrease 800 g by 25% = 600 g

Mass of soil for 2nd flowerpot: increase 3.2 kg by 40% = 4.48 kg

Mass of 2nd flowerpot when full of soil = 600 g + 4.48 kg = 5.08 kg *or* 5080 g

Topic 3

Rounding and error intervals

Key points

- To round a number to a given degree of accuracy, look at the digit to the right of the place value you're rounding to. If that digit is 5 or greater, you round up; if it's less than 5 you round down.

- The first decimal place (sometimes written as *dp*) is the first digit after the decimal point; the second decimal place is the second digit after the decimal point; the third decimal place is the third digit after the decimal point, etc.

- The first significant figure (sometimes written as *sig fig* or *sf*) is the first digit in a number that is not 0; the second significant figure is the next digit after the first significant figure (even if it is 0); the third significant figure is the next digit after the second significant figure (even if it is 0), etc.

- An error interval shows the range of possible values a number could have been <u>before</u> it was rounded. It is shown using inequality symbols in the format:
  Lower Bound ≤ Original number < Upper Bound
  The lower inequality symbol is always ≤ because the lower bound will round to give the required value, whereas the upper inequality symbol is always < because the upper bound is the first value that will <u>not</u> round to give the required value.

Examples

Example 1   Round 27.9468 to 1 dp

The digit in the 1ˢᵗ decimal place is 9	**1** Identify the digit in the place value you are rounding to
The digit to the right of the 1ˢᵗ decimal place is 4	**2** Identify the digit to the right of the place value you are rounding to
The 4 causes the 9 to round down (i.e. stay the same)	**3** Determine whether you should round up or down
The rounded value is 27.9	**4** Any digits to the left of the place value you are rounding to stay the same; any to the right of it are removed (or change to 0 if they are needed to maintain place value)

Example 2   Round 582.0967 to 2 dp

The digit in the 2nd decimal place is 9	**1**	Identify the digit in the place value you are rounding to
The digit to the right of the 2nd decimal place is 6	**2**	Identify the digit to the right of the place value you are rounding to
The 6 causes the 9 to round up to 10	**3**	Determine whether you should round up or down
The rounded value is 582.10	**4**	Any digits to the left of the place value you are rounding to stay the same; any to the right of it are removed (or change to 0 if they are needed to maintain place value)

Example 3   Round 407.63 to 2 sig fig

The 2nd significant figure is 0	**1**	Identify the digit in the place value you are rounding to
The digit to the right of the 2nd significant figure is 7	**2**	Identify the digit to the right of the place value you are rounding to
The 7 causes the 0 to round up to 1	**3**	Determine whether you should round up or down
The rounded value is 410	**4**	Any digits to the left of the place value you are rounding to stay the same; any to the right of it are removed (or change to 0 if they are needed to maintain place value)

Example 4   Round 0.000080216 to 3 sig fig

The 3rd significant figure is 2	**1**	Identify the digit in the place value you are rounding to
The digit to the right of the 3rd significant figure is 1	**2**	Identify the digit to the right of the place value you are rounding to
The 1 causes the 2 to round down (i.e. stay the same)	**3**	Determine whether you should round up or down
The rounded value is 0.0000802	**4**	Any digits to the left of the place value you are rounding to stay the same; any to the right of it are removed (or change to 0 if they are needed to maintain place value)

Example 5   A number, x, has been rounded to the nearest 10 and the result is 90.
Write down the error interval for x.

The rounding unit is 10	**1** Identify the rounding unit (this will be the difference between the lower and upper bounds)
$10 \div 2 = 5$	**2** Halve the rounding unit (because the rounded number is always halfway between the upper and lower bounds)
Lower bound = 90 – 5 = 85 Upper bound = 90 + 5 = 95	**3** To find the lower and upper bounds, subtract/add half of the rounding unit from the rounded number
Error interval is $85 \leq x < 95$	**4** Complete the error interval using the lower and upper bounds

Example 6   A number, y, has been rounded to 1 decimal place and the result is 7.6.
Write down the error interval for y.

The rounding unit is 0.1	**1** Identify the rounding unit (this will be the difference between the lower and upper bounds)
$0.1 \div 2 = 0.05$	**2** Halve the rounding unit (because the rounded number is always halfway between the upper and lower bounds)
Lower bound = 7.6 – 0.05 = 7.55 Upper bound = 7.6 + 0.05 = 7.65	**3** To find the lower and upper bounds, subtract/add half of the rounding unit from the rounded number
Error interval is $7.55 \leq y < 7.65$	**4** Complete the error interval using the lower and upper bounds

Exercise 3

1) a) Round 5483 to the nearest thousand.
   b) Round 5483 to the nearest hundred.
   c) Round 5483 to the nearest ten.

2) a) Round 178.5296 to the nearest whole number.
   b) Round 178.5296 to 1 decimal place.
   c) Round 178.5296 to 2 decimal places.
   d) Round 178.5296 to 3 decimal places.

3) a) Round 85.30647 to 1 significant figure.
   b) Round 85.30647 to 2 significant figures.
   c) Round 85.30647 to 3 significant figures.
   d) Round 85.30647 to 4 significant figures.
   e) Round 85.30647 to 5 significant figures.
   f) Round 85.30647 to 6 significant figures.

4) a) Round 0.00481097 to 1 significant figure.
   b) Round 0.00481097 to 2 significant figures.
   c) Round 0.00481097 to 3 significant figures.
   d) Round 0.00481097 to 4 significant figures.
   e) Round 0.00481097 to 5 significant figures.

5) The mass of a coin is 8 grams to the nearest gram.
   Complete the error interval for the true mass of the coin.
   ……… g $\leq$ mass $<$ ……… g

6) The distance between two cities is 900 km to the nearest 100 km.
   Complete the error interval for the true distance between the cities.
   ……… km $\leq$ distance $<$ ……… km

7) Frank rounds a number, x, to the nearest ten.
   His result is 20
   Complete the error interval for x.
   ……… $\leq x <$ ………

8) Lily rounds a number, y, to the nearest whole number.
   The result is 4
   Complete the error interval for y.
   ……… $\leq y <$ ………

9) Tia rounds a number, z, to 1 decimal place.
   Her result is 6.7
   Complete the error interval for z.
   ……… $\leq z <$ ………

1) a) 5000

   b) 5500

   c) 5480

2) a) 179

   b) 178.5

   c) 178.53

   d) 178.530

3) a) 90

   b) 85

   c) 85.3

   d) 85.31

   e) 85.306

   f) 85.3065

4) a) 0.005

   b) 0.0048

   c) 0.00481

   d) 0.004811

   e) 0.0049110

5) $7.5 \text{ g} \leq \text{ mass } < 8.5 \text{ g}$

6) $850 \text{ km} \leq \text{ distance } < 950 \text{ km}$

7) $15 \leq x < 25$

8) $3.5 \leq y < 4.5$

9) $6.65 \leq z < 6.75$

Rearranging equations

Key points

- To change the subject of a formula, use inverse operations to get the terms containing the subject on one side and everything else on the other side.

Examples

Example 1   Make b the subject of the formula $a = b - 8$

$a = b - 8$	
$a + 8 = b$	**1** Add 8 to both sides of the formula.
$b = a + 8$	**2** You could swap the sides so the subject comes first (you don't *have* to).

Example 2   Make *h* the subject of the formula $k = 3h$

$k = 3h$	
$\frac{k}{3} = h$	**1** Divide throughout by 3.
$h = \frac{k}{3}$	**2** You could swap the sides so the subject comes first (you don't *have* to)

Example 3   Make *t* the subject of the formula $v = u + at$.

$v = u + at$	**1** Subtract *u* from both sides.
$v - u = at$	
$\frac{v-u}{a} = t$	**2** Divide throughout by *a*.
$t = \frac{v-u}{a}$	**3** You could swap the sides so the subject comes first (you don't *have* to).

Example 4 Make n the subject of the formula $S = \frac{n}{2}(a + l)$

$S = \frac{n}{2}(a + l)$	
$2S = n(a + l)$	**1** Multiply throughout by 2.
$\frac{2S}{a+l} = n$	**2** Divide throughout by $(a + l)$.
$n = \frac{2S}{a+l}$	**3** You could swap the sides so the subject comes first (you don't *have* to).

Exercise 4

Change the subject of each formula to the letter given in the brackets.

1 $k = m + g$ [g]   **2** $C = \pi d$ [d]   **3** $P = 2l + 2w$ [w]

4 $x = a + (n - 1)d$ [a]   **5** $D = \dfrac{S}{T}$ [T]   **6** $x = \dfrac{b - c}{d}$ [b]

7 **a** Make A the subject in the formula $\cos(\theta) = \frac{A}{H}$

   b Make H the subject of the formula $\cos(\theta) = \frac{A}{H}$

8 Make r the subject of the following formulae.

   a $C = 2\pi r$ **b** $A = \pi r^2$ **c** $V = \dfrac{4}{3}\pi r^3$ **d** $V = \dfrac{2}{3}\pi r^2 h$

ANSWERS 4

1 $g = k - m$

2 $d = \dfrac{C}{\pi}$

3 $w = \dfrac{P - 2l}{2}$

4 $a = x - (n - 1)d$

5 $T = \dfrac{S}{D}$

6 $b = dx + c$

7   **a** $A = H \times \cos(\theta)$

    b $H = \dfrac{A}{\cos(\theta)}$

8   **a** $r = \dfrac{C}{2\pi}$
      b $r = \sqrt{\dfrac{A}{\pi}}$

    c $r = \sqrt[3]{\dfrac{3V}{4\pi}}$
      d $r = \sqrt{\dfrac{3V}{2\pi h}}$

Solve linear equations

Key points

- To solve an equation means to find the value of the variable that makes the equation true.
- To solve an equation, use inverse operations to simplify the equation until you have the value of the variable.

Examples

Example 1 Solve $2x - 5 = 7$

$2x - 5 = 7$ $2x = 12$ $x = 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 2 Solve $2 - 5x = -8$

$2 - 5x = -8$ $-5x = -10$ $x = 2$	**1** Subtract 2 from both sides. **2** Divide both sides by -5.

Example 3 Solve $4(x - 2) = 3(9 - x)$

$4(x - 2) = 3(9 - x)$ $4x - 8 = 27 - 3x$ $7x - 8 = 27$ $7x = 35$ $x = 5$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

1 Solve these equations.

    a $x + 47 = 31$ **b** $8x - 7 = 3$ **c** $13 = 5 - \frac{x}{4}$

    d $\frac{3x}{5} + 2 = 2.9$ **e** $\frac{4x + 5}{3} = 6$ **f** $3x + 4(5x - 7) = -120$

2 Solve these equations.

    a $9x - 10 = 7x + 24$ **b** $34 - 12x = 28x - 36$

    c $5(2x + 9) - 2(x + 11) = 3(3x + 4) + 46$

3 A rectangular field has a perimeter of 150 m.
    The field is 12 metres longer than it is wide.
    The width of the field is w metres.
    a Write down an equation, in terms of w, to show this information
    b Solve your equation to find the width of the field
    c Calculate the area of the field.

4 Sam thinks of a number, he calls it x.
    Sam adds 3 to his number, then multiplies the result by 6.
    The answer is now 8 times his original number.
    a Write down an equation, in terms of x, to show this information
    b Solve your equation to find the number Sam thought of.

1   **a** $x = -16$ **b** $x = 1.25$ **c** $x = -32$

      d $x = 1.5$ **e** $x = 3.25$ **f** $x = -4$

2   **a** $x = 17$ **b** $x = 1.75$ **c** $x = -35$

3   **a** $4w + 24 = 150$

      b $w = 31.5$

          The field is 31.5 metres wide

      c $A = 31.5 \times (31.5 + 12)$

             $= 1370.25$

          The area of the field is 1370.25 m^2

4   **a** $6(x + 3) = 8x$

      b $x = 9$

          The number Sam thought of was 9.

Solve linear inequalities

A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

Example 1   Solve $2x - 5 < 7$

$2x - 5 < 7$ $\quad 2x < 12$ $\quad\; x < 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 2   Solve $2 - 5x \geq -8$

$2 - 5x \geq -8$ $\quad -5x \geq -10$ $\quad\;\; x \leq 2$	**1** Subtract 2 from both sides. **2** Divide both sides by -5. $\quad$ Remember to reverse the inequality $\quad$ when dividing by a negative $\quad$ number.

Example 3   Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $\quad 4x - 8 > 27 - 3x$ $\quad 7x - 8 > 27$ $\quad\quad 7x > 35$ $\quad\quad\; x > 5$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

1 Solve these inequalities.

  a $4x > 16$ **b** $5x - 7 \le 3$ **c** $1 \ge 3x + 4$

  d $5 - 2x < 12$ **e** $\dfrac{x}{2} \ge 5$ **f** $8 < 3 - \dfrac{x}{3}$

2 Solve these inequalities.

  a $3t + 1 < t + 6$ **b** $2(3n - 1) \ge n + 5$ **c** $3(2 - x) > 2(4 - x) + 4$

3 The shapes shown are both **regular**.
The length of one of their sides is shown, in terms of x.
The perimeter of the pentagon is greater than the perimeter of the triangle.
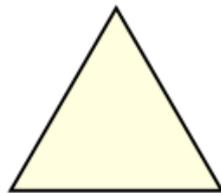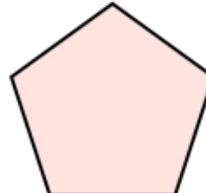
$x + 6$ $x - 2$

  a Write down an inequality, in terms of x to show this information
  b Solve your inequality to find the possible range of values for x.

Extension

4 Find the range of values of x which satisfies **both**

$$3(x + 2) \le 30 \quad and \quad 4x + 3 > 21$$

1 a $x > 4$ b $x \leq 2$ c $x \leq -1$

   d $x > -3.5$ e $x \geq 10$ f $x < -15$

2 a $t < 2.5$ b $n \geq 1.4$ c $x < -6$

3 a $3(x + 6) < 5(x - 2)$ b $x > 14$

4 $3(x + 2) \leq 30$ $4x + 3 > 21$
   $x \leq 8$ $x > 4.5$

   i.e. $4.5 < x \leq 8$

Topic 6a

Straight line graphs

Key points

- A horizontal line has the equation $y = a$, where a is the y-intercept.
- A vertical line has the equation $x = b$, where b is the x-intercept.
- A diagonal line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept.
- The gradient of a line describes how steep it is. If the gradient is negative the line slopes downwards, if the gradient is positive the line slopes upwards (from left-right). The value of the gradient tells you the vertical change if the x-value is increased by 1.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
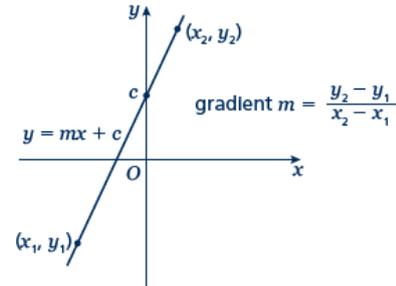
Examples

Example 1 A vertical line passes through the point (3, 1). Write the equation of the line.
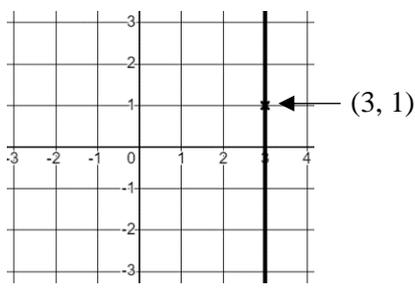
(3, 1) x-intercept $= 3$ So $x = 3$	**1** A vertical line has equation $x = b$, where b is the x-intercept.

Example 2   A straight line passes through the points (-2, 4) and (3, 4). Write the equation of the line.
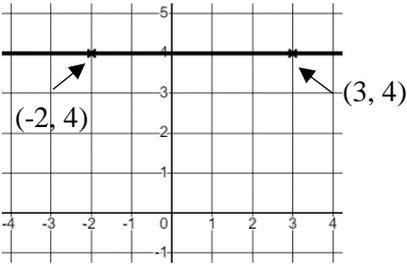
Line is horizontal y-intercept $= 4$ So $y = 4$	**1** A horizontal line has equation $y = a$, where a is the y-intercept.

Example 3   A straight line has gradient $-\dfrac{1}{2}$ and y-intercept 3.

Write the equation of the line in the form $y = mx + c$.

$m = -\dfrac{1}{2}$ and $c = 3$ So $y = -\dfrac{1}{2}x + 3$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation.

Example 4   Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \dfrac{2}{3}x - \dfrac{4}{3}$ Gradient $= m = \dfrac{2}{3}$ y-intercept $= c = -\dfrac{4}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \ldots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 5 Find the equation of the line shown.
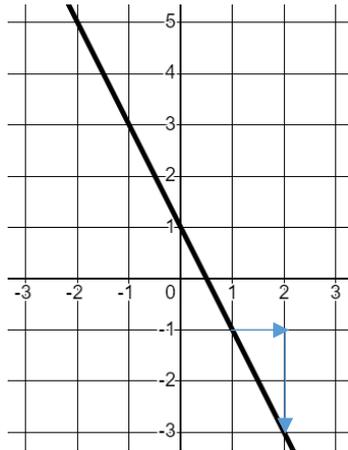
y-intercept is 1 So $c = 1$	**1** Find the y-intercept of the line
Gradient is -2 So $m = -2$	**2** Find the gradient of the line
So $y = -2x + 1$	**3** Substitute $c = 1$ and $m = -2$ into the equation $y = mx + c$

Exercise 6a

1 Find the gradient and the y-intercept of the following equations.

a $y = 3x + 5$

b $y = -\dfrac{1}{2}x - 7$

c $2y = 4x - 3$

d $2x - 3y - 7 = 0$

> **Hint**
> Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	y-intercept	Equation of the line
5	0	
–3	2	
4	–7	

3 Write the equation of each of the following lines

(a)
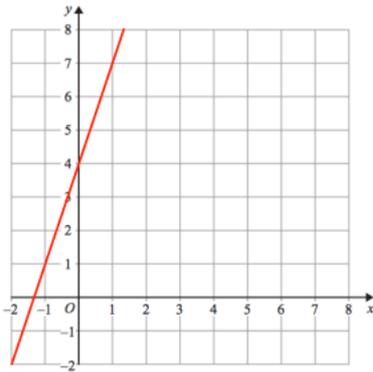
(b)
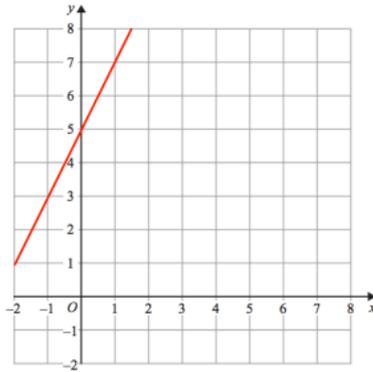
(c)
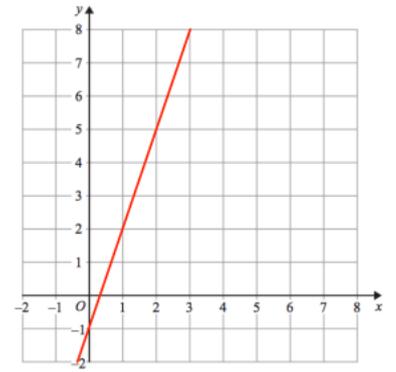
(d)
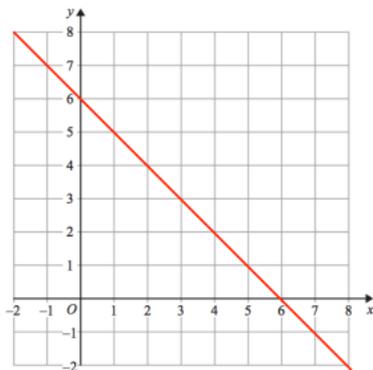
(e)
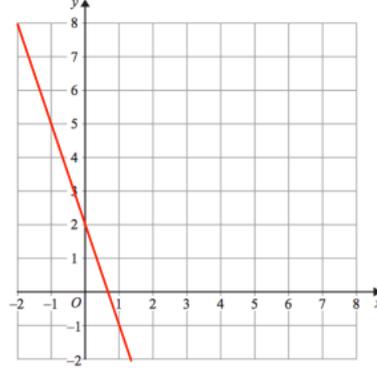
(f)

4 Write the equation of each of the following lines

(a)

B (3, 11)

A (0, 5)

(b)

A (0, 8)

B (8, 4)
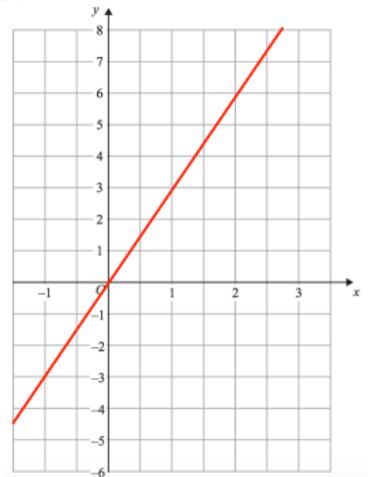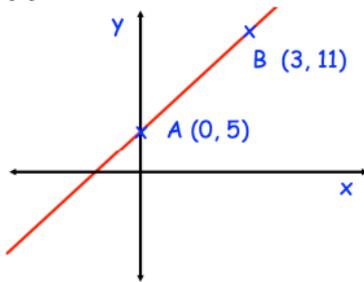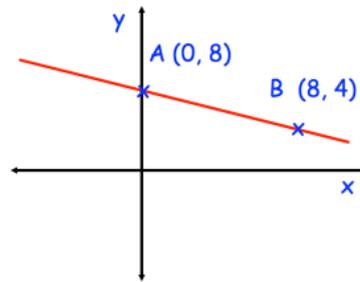
1 **a** $m = 3, c = 5$ **b** $m = -\dfrac{1}{2}, c = -7$

   c $m = 2, c = -\dfrac{3}{2}$ **d** $m = \dfrac{2}{3}, c = -\dfrac{7}{3}$ or $-2\dfrac{1}{3}$

2

Gradient	*y*-intercept	Equation of the line
5	0	$y = 5x$
–3	2	$y = -3x + 2$
4	–7	$y = 4x - 7$

3 **a** $y = 3x + 4$ **b** $y = 2x + 5$ **c** $y = 3x - 1$

   e $y = -x + 6$ **f** $y = -3x + 2$ **g** $y = 1.5x$

4 **a** $y = 3x + 5$ **b** $y = -0.5x + 8$

Parallel and perpendicular lines

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- If two lines are parallel, they have the same gradient.
- If two lines are perpendicular, the product of their gradients is -1. This means that the two gradients are the negative reciprocals of each other, i.e. if the gradient of one of the lines is m then the gradient of the second line is $\frac{-1}{m}$.

Parallel Lines

90°
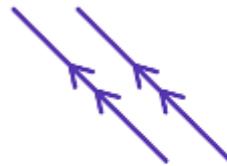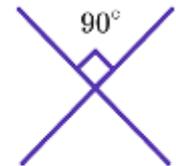
Perpendicular Lines

Examples

Example 1   Write down the equation of the line that is parallel to $y = 3x + 1$ and passes through the point (0, 7)

$y = 3x + 1$	**1** The gradient of this line is 3.
(0, 7) is on the y-axis	**2** The y-intercept of the line is 7.
$y = 3x + 7$	**3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 2   Write down the equation of the line that is parallel to $2y + 8x = 14$ and passes through the point (0, -3)

$2y + 8x = 14$	**1** Rearrange into the form $y = mx + c$
$y = -4x + 7$	**2** The gradient of the line is - 4.
(0, -3) is on the y-axis	**3** The y-intercept of the line is - 3.
$y = -4x - 3$	**4** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Write down the equation of the line that is perpendicular to $y = 3x + 1$ and passes through the point $(0, 5)$

$y = 3x + 1$	**1** The gradient of this line is 3.
Gradient $= -\frac{1}{3}$	**2** The gradient of the required line is the negative reciprocal of 3.
$(0, 5)$ is on the y-axis	**3** The y-intercept of the line is 5.
$y = -\frac{1}{3}x + 5$	**3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 4 Write down the equation of the line that is perpendicular to $y = 0.5x - 5$ and passes through the point $(0, 8)$

$y = 0.5x - 5$	**1** The gradient of this line is 0.5 or $\frac{1}{2}$.
Gradient $= -2$	**2** The gradient of the required line is the negative reciprocal of $\frac{1}{2}$.
$(0, 8)$ is on the y-axis	**3** The y-intercept of the line is 8.
$y = -2x + 5$	**4** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 5 Write down the equation of the line that is perpendicular to $y = -3.5x + 2$ and passes through the point $(0, -8)$

$y = -3.5x + 2$	**1** The gradient of this line is -3.5 or $-\frac{7}{2}$.
Gradient $= \frac{2}{7}$	**2** The gradient of the required line is the negative reciprocal of $-\frac{7}{2}$.
$(0, -8)$ is on the y-axis	**3** The y-intercept of the line is -8.
$y = \frac{2}{7}x - 8$	**4** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

1 Write down the equation of each of the following lines

    a parallel to $y = 3x - 5$ and passing through $(0, 2)$

    b parallel to $y = -5x - 8$ and passing through $(0, 6)$

    c parallel to $y = -0.5x + 10$ and passing through the origin

    d parallel to $x + y = 2$ and passing through $(0, -4)$

    e parallel to $x - 2y + 3 = 0$ and passing through $(0, 5)$

2 Write down the equation of the line parallel to line 1 and passing through point A

    a   **b**
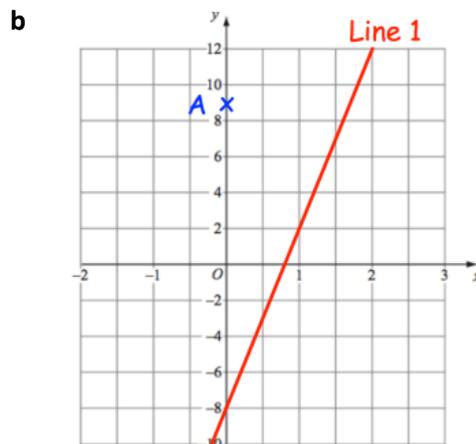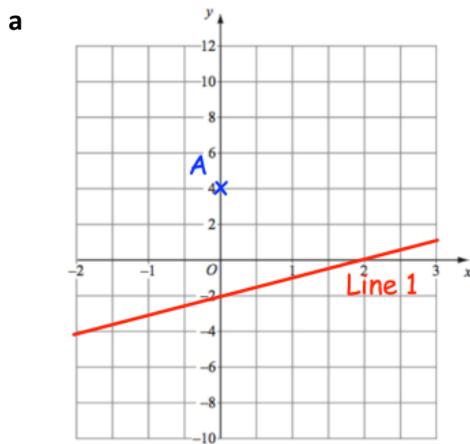
HINT: Check the scales on the axes carefully

3 Write down the equation of each of the following lines

    a perpendicular to $y = -3x - 8$ and passing through $(0, 2)$

    b perpendicular to $y = \frac{1}{3}x + 5$ and passing through the origin

    c perpendicular to $y = -\frac{2}{9}x - 1$ and passing through $(0, -4)$

    d perpendicular to $y = 2\frac{3}{4}x + 9$ and passing through $(0, 3)$

    e perpendicular to $5x - 3y + 3 = 0$ and passing through $(0, -1)$

4 Write down the equation of the line perpendicular to line 1 and passing through point A

a

b
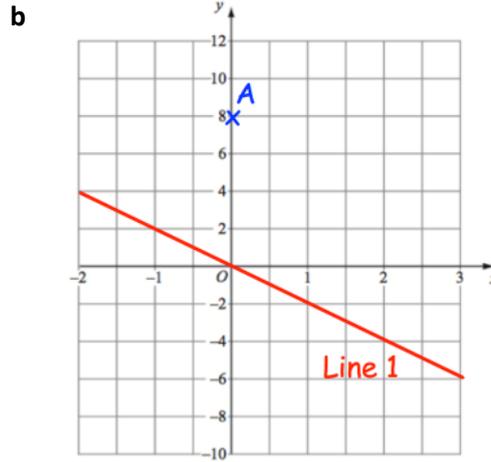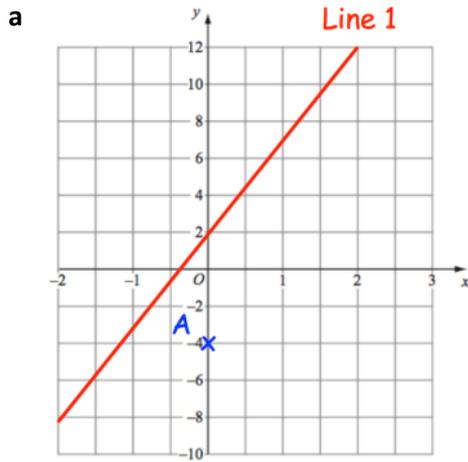
HINT: Check the scales on the axes carefully

Extension

5 Two straight lines are shown.

Line 1 has equation $y = \dfrac{3}{2}x - 24$

a Find the equation of Line 2

b Are the lines perpendicular? Show your working to justify your answer.

1 **a** $y = 3x + 2$ **b** $y = -5x + 6$

   c $y = -0.5x$ **d** $y = -x - 4$

   e $y = 0.5x + 5$

2 **a** $y = x + 4$ **b** $y = 10x + 9$

3 **a** $y = \frac{1}{3}x + 2$ **b** $y = -3x$

   c $y = \frac{9}{2}x - 4$ **d** $y = -\frac{4}{11}x + 3$

   e $y = -\frac{3}{5}x - 1$

4 **a** $y = -\frac{1}{5}x - 4$ **b** $y = \frac{1}{2}x + 8$

5

   a $y = -\frac{6}{10}x + 12$

   b If the lines were perpendicular, the gradient of Line 2 would be $-\frac{2}{3}$. As $-\frac{6}{10}$ is not equivalent to $-\frac{2}{3}$, the lines are not perpendicular.
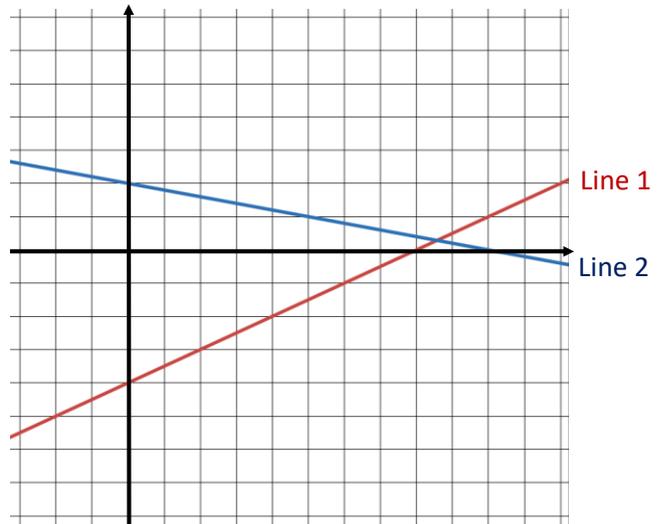
Topic 7a

Area of 2D shapes and surface area of 3D shapes

Key points

- The area of a 2D shape tells you how much space there is inside it.
- To calculate the area of a 2D shape use the relevant formula:

Shape	Formula	Notes
Rectangle	Area = bh	b is the base and h is the height
Parallelogram	Area = bh	b is the base and h is the perpendicular height
Triangle	Area = $\frac{1}{2}(bh)$	b is the base and h is the perpendicular height
Trapezium	Area = $\frac{1}{2}(a+b)h$	a and b are the parallel sides and h is the perpendicular height
Circle	Area = πr^2	r is the radius

- To find the area of a compound shape, split it into more simple shapes then combine their areas.
- The surface area of a 3D shape is the total area of all of its faces. To calculate it, find the area of each face, then add these together.

Examples

Example 1   Calculate the area of the trapezium

9 cm

4 cm

3 cm
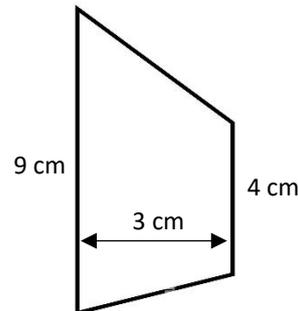
Area = $\frac{1}{2}(a+b)h$ $= \frac{1}{2}(9+4) \times 3$ $= 19.5 \text{ cm}^2$	**1** The parallel sides are 9 cm and 4 cm, the perpendicular 'height' is 3 cm **2** Units of area are square units

Example 2   Calculate the area of the circle, giving your answer correct to 3 significant figures.

12 mm

Area $= \pi r^2$	**1** The diameter is 12 mm so the radius is 6 mm
$= \pi \times 6^2$	
$= 113.097\ 335\ldots$	**2** Round to 3 significant figures
$= 113\ \text{mm}^2$	

Example 3   Calculate the area of the shaded shape shown.
The dashed lines show a square.
Give your answer correct to 3 significant figures.

8 cm

$Area_{square} = b^2$	**1** Start by finding the total area
$= 8^2$	
$= 64\ \text{cm}^2$	
$Area_{circle} = \pi r^2$	**2** The 4 sectors would combine to make a complete circle
$= \pi \times 4^2$	
$= 16\pi$	
$Area_{shaded} = 64 - 16\pi$	**3** Subtract the area of the 4 sectors from the overall square (use exact value for as long as possible)
$= 13.734\ 517\ldots$	
$= 13.7\ \text{cm}^2$	**4** Round to 3 significant figures

Example 4   The scalene triangle shown has an area of 33.6 m^2.
Calculate the length of *b*.

10 m   6 m   *b*

Area $= \frac{1}{2}bh$	**1** The area is 33.6 m^2 and the height is 6 m
$33.6 = \frac{1}{2} \times b \times 6$	**2** Simplify the equation
$33.6 = 3b$	**3** Solve the equation for *b*
$b = 11.2\ \text{m}$	

Example 5   Calculate the surface area of the triangular prism shown.

$Area_{front} = \frac{1}{2}bh$	**1** The front face is a triangle
$= \frac{1}{2} \times 8 \times 3$	
$= 12 \text{ cm}^2$	
$Area_{right} = bh$	**2** The right-hand face is a rectangle
$= 5 \times 12$	
$= 60 \text{ cm}^2$	
$Area_{base} = bh$	**3** The face on the base is a rectangle
$= 8 \times 12$	
$= 96 \text{ cm}^2$	
Surface area $= 12 \times 2 + 60 \times 2 + 96$	**4** The back face is identical to the front and the left-hand face is identical to the right so do not need to be calculated individually
$= 240 \text{ cm}^2$	**5** Units of area are square units

1 Farmer Martin keeps chickens in the field below.
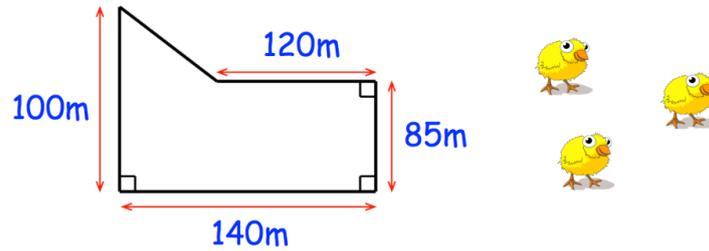
a Calculate the area of the field

b Each chicken needs 3 m^2.
What is the maximum number of chickens Farmer Martin can keep in this field?

2 The total area of the rectilinear shape shown is 151 cm^2.
Calculate the length of x.
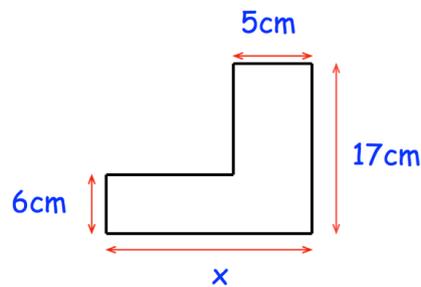
3 A circular disk of diameter 6 cm is cut from a square piece of metal of side length 6 cm.
What percentage of the metal is wasted (the wasted metal is shown shaded on the diagram below)?
Give your answer correct to 3 significant figures.
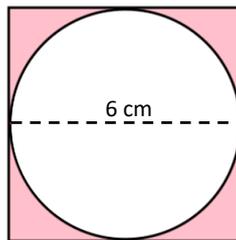
4 The diagram shows a prism.
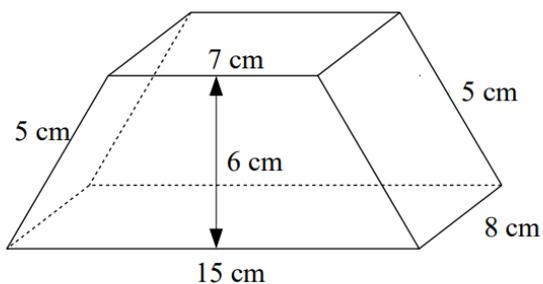
The cross section of the prism is in the shape of a trapezium.

Calculate the total surface area of the prism.

1 **a**
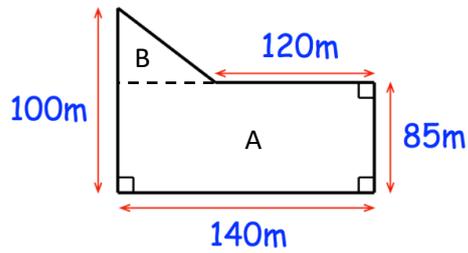
$Area_A = 11900 \text{ m}^2$
$Area_B = 150 \text{ m}^2$

Total area = 12050 m²

b 12050 ÷ 3 = 4016.666…

i.e. Farmer Martin can keep a maximum of 4016 chickens in this field.

2 $x = 16$ cm

3 $Area_{square} = 36 \text{ cm}^2$
$Area_{circle} = 9\pi$
$Area_{wasted} = 36 - 9\pi$

Percentage wasted $= \dfrac{36 - 9\pi}{36} \times 100 = 21.5\%$ (3 sig fig)

4 $Area_{front} = 66 \text{ cm}^2$
$Area_{top} = 56 \text{ cm}^2$
$Area_{base} = 120 \text{ cm}^2$
$Area_{right} = 40 \text{ cm}^2$

Surface area = 66 + 56 + 120 + 2 × 40 = 322 cm²

Volume of 3D prisms

Key points

- The volume of a 3D shape tells you how much space there is inside it.
- A prism is a 3D shape that has a constant cross-section.
- To calculate the volume of a prism:
    1. Find the area of its cross-section (CSA)
    2. Multiply the CSA by the prism's length

Triangular Prism   **Pentagonal Prism**

Hexagonal Prism

Examples

Example 1   Calculate the volume of the triangular prism

11cm   9cm   15cm   14cm
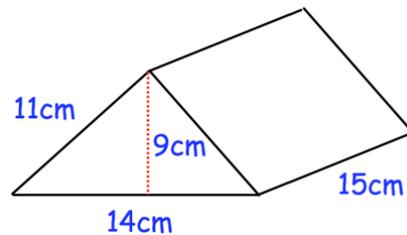
Volume = CSA × length $= \frac{1}{2}(14 \times 9) \times 15$ $= 945 \text{ cm}^3$	**1** The cross-section is a triangle **2** Units of volume are cubed units

Example 2   Calculate the volume of the cylinder.
Give your answer correct to 3 significant figures.

4cm   3cm

Volume = CSA × length $= \pi \times 4^2 \times 3$ $= 150.796\ 447\ldots$ $= 151 \text{ cm}^3$	**1** The cross-section is a circle **2** Round to 3 significant figures **3** Units of volume are cubed units

**1**   Calculate the volume of each prism

a

b
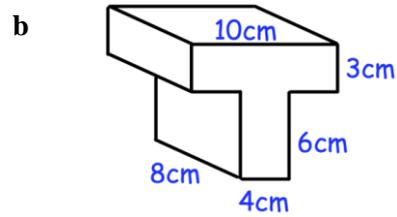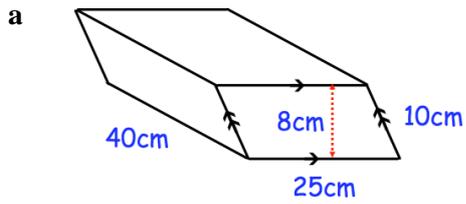
2   The solid triangular prism shown below is made from metal.
The prism is melted down and the metal is used to make a number of cubes of side length 4 cm.
What is the maximum number of cubes that can be made?
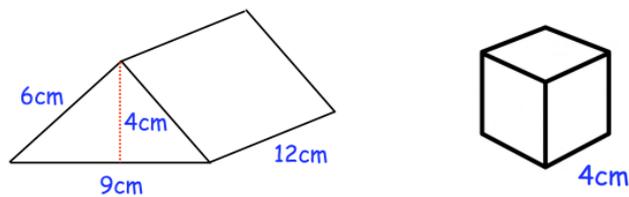
3   The cylindrical vase shown below is full to the brim with water.
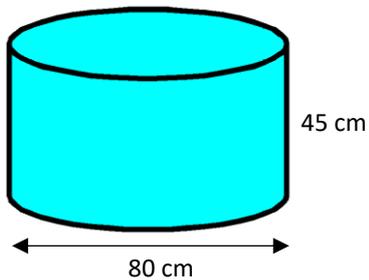
The water is poured from the vase into the empty cuboid-shaped water tank shown below.
None of the water is spilled during this process.
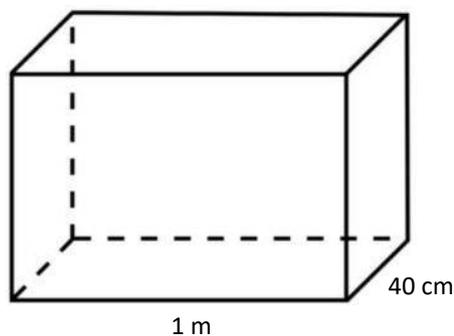
How deep will the water be in the water tank?
Give your answer correct to 3 significant figures.

1 **a**   Volume = 8000 cm^3 **b**   Volume = 432 cm^3

2 $Volume_{triangular\ prism} = 216$ cm^3
  $Volume_{cube} = 64$ cm^3

  $216 \div 64 = 3.375$
  i.e. a maximum of 3 cubes can be made

3 $Volume_{water} = 72000\pi$
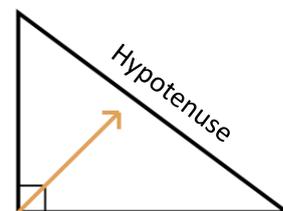 Cross-section of tank = $100 \times 40 = 4000$ cm^2

  $72000\pi \div 4000 = 56.548\ 667…$
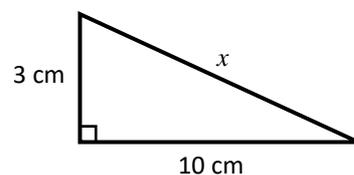 i.e. water depth = 56.5 cm (3 sig fig)

Pythagoras' Theorem

Key points

- In a right-angled triangle the side opposite the right angle is called the hypotenuse. This is always the longest side.
- Pythagoras' Theorem states: $a^2 + b^2 = c^2$
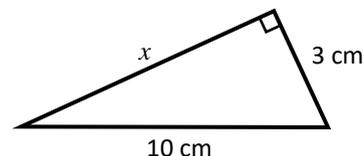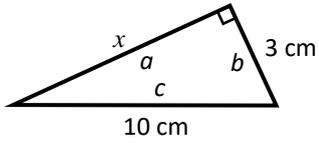 Where c is the length of the hypotenuse and a and b are the lengths of the other two sides.

Examples

Example 1   Calculate the length of side x.
Give your answer correct to 3 significant figures.

$a^2 + b^2 = c^2$	**1** Always start by labelling the sides, ensuring the hypotenuse is c.
$3^2 + 10^2 = x^2$	**2** Substitute the lengths into Pythagoras' Theorem.
$9 + 100 = x^2$	**3** Solve for x.
$109 = x^2$	
$x = \sqrt{109}$	
$= 10.440\ 306\ldots$	
$= 10.4$ cm	**4** Round your answer to 3 significant figures and write the units in your answer.

Example 2   Calculate the length of side x.
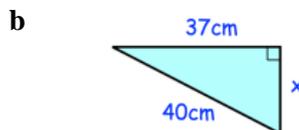Give your answer correct to 3 significant figures.

	1 Always start by labelling the sides, ensuring the hypotenuse is c.
$a^2 + b^2 = c^2$	**2** Substitute the lengths into Pythagoras' Theorem.
$x^2 + 3^2 = 10^2$	**3** Solve for x.
$x^2 + 9 = 100$	
$x^2 = 100 - 9$	
$x^2 = 91$	
$x = \sqrt{91}$	
$= 9.539\ 392\ldots$	**4** Round your answer to 3 significant figures and write the units in your answer.
$= 9.54$ cm	

Exercise 8a

1   Calculate the value of x, giving your answers correct to 3 significant figures where appropriate

a   **b**

2   Calculate the value of x, giving your answers correct to 3 significant figures where appropriate

a   **b**
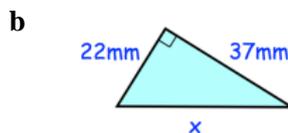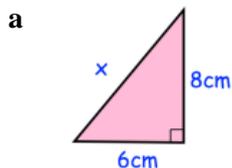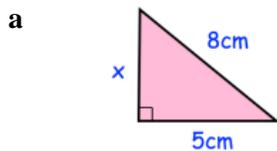
3   From Alton airfield, a plane flies due North for 52 miles, then due East for 186 miles and lands at Brightwater airbase.

Calculate the direct distance from Alton airfield to Brightwater airbase, giving your answer correct to 3 significant figures.

ANSWERS 8a

1 **a** $x = 10$ cm **b** $x = 43.0$ mm

2 **a** $x = 6.24$ cm **b** $x = 15.2$ cm

3 Distance = 193 miles

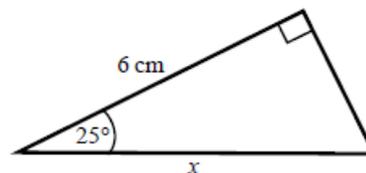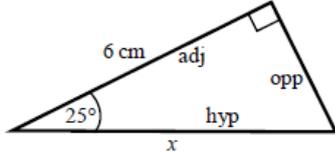# Trigonometry in right-angled triangles

Key points

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle θ is called the opposite
  - the side next to the angle θ is called the adjacent.
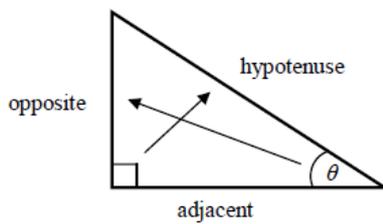
- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle θ, $\quad \sin \theta = \dfrac{\text{opp}}{\text{hyp}}$

  - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ, $\quad \cos \theta = \dfrac{\text{adj}}{\text{hyp}}$

  - the ratio of the opposite side to the adjacent side is the tangent of angle θ, $\quad \tan \theta = \dfrac{\text{opp}}{\text{adj}}$

- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.
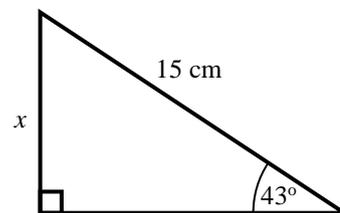
Examples

Example 1   Calculate the length of side x.
Give your answer correct to 3 significant figures.

$\cos \theta = \dfrac{\text{adj}}{\text{hyp}}$	**1** Always start by labelling the sides.
	2 You are given the adjacent and the hypotenuse so use the cosine ratio.
$\cos 25° = \dfrac{6}{x}$	**3** Substitute the sides and angle into the cosine ratio.
$x = \dfrac{6}{\cos 25°}$	**4** Rearrange to make x the subject.
$x = 6.620\ 267\ 5...$	**5** Use your calculator to work out $6 \div \cos 25°$.
$x = 6.62$ cm	**6** Round your answer to 3 significant figures and write the units in your answer.

Example 2   Calculate the length of side x.
Give your answer correct to 3 significant figures.

	1 Always start by labelling the sides.
$\sin \theta = \frac{opp}{hyp}$	**2** You are given the opposite and the hypotenuse so use the sine ratio.
$\sin 43° = \frac{x}{15}$	**3** Substitute the sides and angle into the sine ratio.
$x = 15 \times \sin 43°$	**4** Rearrange to make x the subject and calculate.
$x = 10.229\ 975\ 4\ldots$	
$x = 10.2$ cm	**5** Round your answer to 3 significant figures and write the units in your answer.

Example 3   Calculate the size of angle x.
Give your answer correct to 3 significant figures.

	1 Always start by labelling the sides.
$\tan \theta = \frac{opp}{adj}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \frac{3}{4.5}$	**3** Substitute the sides and angle into the tangent ratio.
$x = \tan^{-1} \left(\frac{3}{4.5} \right)$	**4** Use $\tan^{-1}$ to find the angle.
$x = 33.690\ 067\ 5...$	**5** Use your calculator to work out $\tan^{-1}(3 \div 4.5)$.
$x = 33.7°$	**6** Round your answer to 3 significant figures and write the units in your answer.

1 Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a

68°   7 cm
x

b

x
30°
8 cm

c

x 6 cm
65°

d

6.2 cm
x
57°
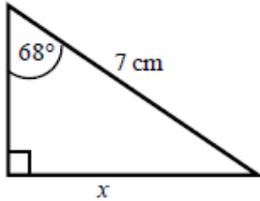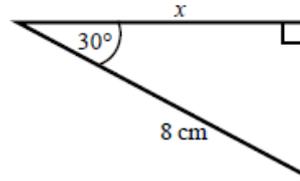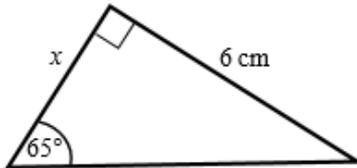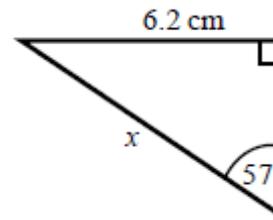
2 Calculate the size of angle x in each triangle.
Give your answers correct to 3 significant figures.

a

5 cm
x
4 cm

b

6.3 cm
7.5 cm
x

c

45 mm   42 mm
x

d

4 cm
x
2.5 cm
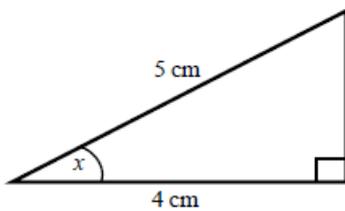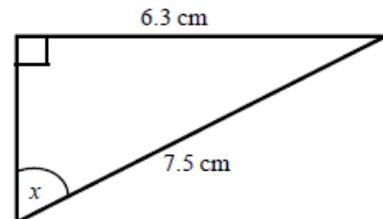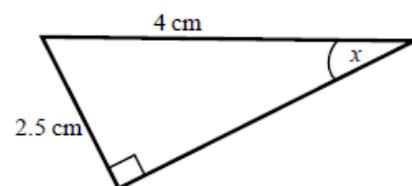
3 The foot of a ladder is on horizontal ground and the top of the ladder is leant against a vertical wall.
The ladder reaches 4.3 metres up the wall and makes an angle of 36° with the wall.

Calculate the length of the ladder, giving your answer correct to 3 significant figures.

1   **a** $x = 6.49$ cm   **b** $x = 6.93$ cm
    c $x = 2.80$ cm   **d** $x = 7.39$ cm

2   **a** $x = 36.9º$ **b** $x = 57.1º$
    c $x = 47.0º$ **d** $x = 38.7º$

3   5.32 metres

Averages and range

Key points

- Averages are a measure of 'central tendency'. There are three types of average:
  - Mean - this would be the value if the total were shared out equally; find it by adding the data values then dividing this by the number of data values
  - Median - this is the value in the middle of an ordered list (if there are an even number of data values in the list, the median is half-way between the two middle data values)
  - Mode - this is the most common data value. A data set that has a single mode is called 'unimodal', a data set with two modes is called 'bimodal', a data set with three modes is called 'trimodal' and a data set with more than three modes is called 'multimodal'.
- The range is a measure of 'spread'; it is the difference between the largest and smallest data value. To calculate the range, use $Range = largest - smallest$

Examples

Example 1   Calculate the mean of the following list of data:

7, 3, 8, 0, 7, 4, -1, 5, 2

Give your answer correct to 3 significant figures.

$Mean = \frac{7 + 3 + 8 + 0 + 7 + 4 + (-1) + 5 + 2}{9}$ $= \frac{35}{9}$ $= 3.888\ 888\ldots$ $= 3.89$	**1** Add the data values then divide by the number of values (9) **2** Round your answer to 3 significant figures

Example 2   Calculate the median of the following list of data:

7, 3, 8, 0, 7, 4, -1, 5, 2

-1, 0, 2, 3, 4, 5, 7, 7, 8 ↑ Median = 4	**1** The data must be arranged in order **2** The median is the value in the middle of the list

Example 3 Calculate the median of the following list of data:

7, 3, 8, 0, 7, 4, -1, 5, 2, 15

-1, 0, 2, 3, 4, 5, 7, 7, 8, 15 Median = 4.5	**1** The data must be arranged in order **2** The median is the value in the middle of the list; because there is an even number of data values, this is half-way between the two middle data values

Example 4 Calculate the mode of the following list of data:

7, 3, 8, 0, 7, 4, -1, 5, 2, 15

-1, 0, 2, 3, 4, 5, 7, 7, 8, 15 Mode = 7	**1** The mode is easiest to identify when the data is arranged in order **2** The mode is the data value that occurs most often

Example 5 Calculate the mode of the following list of data:

7, 3, 8, 0, 7, 4, -1, 5, 2, 15

-1, 0, 2, 3, 4, 5, 7, 7, 8, 15 Range = 15 – (-1) = 16	**1** The range is easiest to identify when the data is arranged in order **2** The range is the difference between the largest and smallest data values

Exercise 9

1   Find the mean, median, mode and range for the following data sets, giving your answers to 3 significant figures where necessary

    a   9, 1, 3, 6, 7, 8, 9   **b**   - 4, 5, - 7, - 1, 2, 9, 2, 5

2   Aarman is conducting an experiment to investigate whether plants grow taller, on average, when fed with Fertilizer A or Fertilizer B. He has grown 10 plants using each fertilizer and has measured their heights; his data is shown below.
Use Aarman's data to write the conclusion for his experiment.

Plant height (cm) Fertilizer A	Plant height (cm) Fertilizer B
30.5	31.6
27.9	33.4
30.1	29.2
31.7	32.0
34.8	31.9
28.6	25.4
25.9	32.8
32.4	32.2
30.7	33.1
31.3	32.9

Conclusion:
According to the data above, on average, plants grow taller when they are fed with Fertilizer A / B.

                                                                (delete as appropriate)

ANSWERS 9

1   **a**   Mean = 6.14   **b**   Mean = 1.38

            Median = 7   Median = 2

            Mode = 9   Mode = 2 and 5 (bimodal)

            Range = 8   Range = 16

2   Fertilizer A: Fertilizer B:

     Mean = 30.4 cm   Mean = 31.5 cm

     Median = 30.6 cm   Median = 32.1 cm

     Mode = no mode   Mode = no mode

Conclusion:
According to the data above, on average, plants grow taller when they are fed with Fertilizer B.

Probability

Key points

- Probability is a measure of how likely an event is to happen.
- It is measured on a scale from 0 to 1, where 0 represents an impossible event and 1 represents an event that is certain.
- If all possible outcomes in an event are equally likely, then the probability of any of them happening can be calculated using $\frac{number\ of\ successful\ outcomes}{total\ number\ of\ outcomes}$
- The sum of the probabilities of all possible outcomes in an event is always 1.

Examples

Example 1   A bag contains 5 red sweets, 8 yellow sweets and 12 green sweets.
One sweet is to be chosen from the bag at random.
Calculate the probability that the sweet chosen is yellow.

Number of 'successful' outcomes is 8 (8 yellow sweets) Total number of sweets/outcomes = 5 + 8 + 12 $\qquad\qquad\qquad = 25$ Probability $= \frac{8}{25}$	**1** Identify the number of 'successful' outcomes **2** Identify the total number of outcomes **3** $Probability = \frac{number\ of\ successful\ outcomes}{total\ number\ of\ outcomes}$ Where selecting a yellow sweet is a 'success'

Example 2   A box contains the following number cards.

1	2	2	3	5	6	7	8	8	10

One card is to be selected from the box at random.
Calculate the probability that the card selected is either an even number or a multiple of 5.

Even cards: 2, 2, 6, 8, 8, 10 Multiples of 5: 5, 10 Number of 'successful' outcomes is 7 (7 cards have either even number or multiple of 5) Total number of cards/outcomes is 10 Probability $= \frac{7}{10}$	**1** Identify the number of 'successful' outcomes, taking care not to double-count any cards (i.e. 10) **2** Identify the total number of outcomes **3** $Probability = \frac{number\ of\ successful\ outcomes}{total\ number\ of\ outcomes}$

Example 3   The table below shows the probability of scoring each value on a biased dice.
Calculate the probability the dice will score 5.

Score	1	2	3	4	5	6
Probability	0.1	0.2	0.15	0.3	x	0.05

$0.1 + 0.2 + 0.15 + 0.3 + x + 0.05 = 1$	**1** The sum of the probabilities of all possible outcomes in an event is always 1
$0.8 + x = 1$	**2** Solve for x
$x = 0.2$	

Exercise 10

1   Kate is going to select one of the shape cards at random.

    a   What is the probability she selects a card with a triangle on it?

    b   What is the probability she selects a card that does **not** have a circle on it?

    c   What is the probability she selects a card with either a triangle or a circle on it?

2   Raymond is going to select one of the number cards at random.

    a   What is the probability he selects a card with a value less than 3?

    b   What is the probability he selects a card with a square number?

    c   What is the probability he selects a card with either a square number **or** an even number?

3   There are only pink, yellow, green and blue counters in a bag.
The table below shows the probability that a counter taken at random from the bag will be pink, green or blue.

Colour	Pink	Yellow	Green	Blue
Probability	0.5		0.1	0.2

    a   Work out the probability that the counter taken is yellow.

    b   There are 40 counters in the bag in total.
Work out the number of blue counters.

4 The two-way table below gives information about 90 people who sat their driving test last week.

	Under 20 driving lessons	20 or over driving lessons	total
Pass		21	30
Fail	45		
total			90

a Complete the two-way table.

b One of the people is selected at random.
Work out the probability this person passed their driving test.

c One of the people who passed their driving test is selected at random.
Work out the probability this person had under 20 driving lessons.

ANSWERS 10

1 **a** $\frac{5}{12}$ **b** $\frac{6}{12}$ or $\frac{1}{2}$ **c** $\frac{11}{12}$

2 **a** $\frac{2}{9}$ **b** $\frac{3}{9}$ or $\frac{1}{3}$ **c** $\frac{6}{9}$ or $\frac{2}{3}$

3 **a** 0.2 **b** 8 blue counters

4 **a**

	Under 20 driving lessons	20 or over driving lessons	total
Pass	9	21	30
Fail	45	15	60
total	54	36	90

b $\frac{30}{90}$ or $\frac{1}{3}$

c $\frac{9}{30}$ or $\frac{3}{10}$