

Maths summer work for

NEW Lower Sixth

The content of this booklet is assumed prior knowledge for all of the Maths courses that run in the Sixth Form. It contains several algebra topics as well as trigonometry, which would have been covered in the GCSE Higher course. A firm grasp of this content is essential in order to be successful at Maths in the Sixth Form.

You will be tested on this content within the first two weeks of starting in September. Other topics that will be on that test, which are also assumed prior knowledge, include indices, surds and inequalities.

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Area of non-right angled triangle

Factorising expressions

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac .
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$$

The highest common factor is $3x^2y$.
So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$$

This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$$b = 3, ac = -10$$

$$\text{So } x^2 + 3x - 10 = x^2 + 5x - 2x - 10$$

$$= x(x + 5) - 2(x + 5)$$

$$= (x + 5)(x - 2)$$

- 1 Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and -2)
- 2 Rewrite the b term ($3x$) using these two factors
- 3 Factorise the first two terms and the last two terms
- 4 $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11, ac = -60$ So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	<ol style="list-style-type: none"> 1 Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) 2 Rewrite the b term ($-11x$) using these two factors 3 Factorise the first two terms and the last two terms 4 $(2x - 5)$ is a factor of both terms
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Example 5 Simplify $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$ For the numerator: $b = -4, ac = -21$ So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$ For the denominator: $b = 9, ac = 18$ So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$ So $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$ $= \frac{x - 7}{2x + 3}$	<ol style="list-style-type: none"> 1 Factorise the numerator and the denominator 2 Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3) 3 Rewrite the b term ($-4x$) using these two factors 4 Factorise the first two terms and the last two terms 5 $(x - 7)$ is a factor of both terms 6 Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3) 7 Rewrite the b term ($9x$) using these two factors 8 Factorise the first two terms and the last two terms 9 $(x + 3)$ is a factor of both terms 10 $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1
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Practice

1 Factorise.

a $6x^4y^3 - 10x^3y^4$

c $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

b $21a^3b^5 + 35a^5b^2$

2 Factorise

a $x^2 + 7x + 12$

c $x^2 - 11x + 30$

e $x^2 - 7x - 18$

g $x^2 - 3x - 40$

b $x^2 + 5x - 14$

d $x^2 - 5x - 24$

f $x^2 + x - 20$

h $x^2 + 3x - 28$

3 Factorise

a $36x^2 - 49y^2$

c $18a^2 - 200b^2c^2$

b $4x^2 - 81y^2$

4 Factorise

a $2x^2 + x - 3$

c $2x^2 + 7x + 3$

e $10x^2 + 21x + 9$

b $6x^2 + 17x + 5$

d $9x^2 - 15x + 4$

f $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a $\frac{2x^2 + 4x}{x^2 - x}$

c $\frac{x^2 - 2x - 8}{x^2 - 4x}$

e $\frac{x^2 - x - 12}{x^2 - 4x}$

b $\frac{x^2 + 3x}{x^2 + 2x - 3}$

d $\frac{x^2 - 5x}{x^2 - 25}$

f $\frac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

a $\frac{9x^2 - 16}{3x^2 + 17x - 28}$

c $\frac{4 - 25x^2}{10x^2 - 11x - 6}$

b $\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

d $\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$

Hint

Take the highest common factor outside the bracket.

Extend

7 Simplify $\sqrt{x^2 + 10x + 25}$

8 Simplify $\frac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

Answers

- 1** **a** $2x^3y^3(3x - 5y)$ **b** $7a^3b^2(3b^3 + 5a^2)$
 c $5x^2y^2(5 - 2x + 3y)$
- 2** **a** $(x + 3)(x + 4)$ **b** $(x + 7)(x - 2)$
 c $(x - 5)(x - 6)$ **d** $(x - 8)(x + 3)$
 e $(x - 9)(x + 2)$ **f** $(x + 5)(x - 4)$
 g $(x - 8)(x + 5)$ **h** $(x + 7)(x - 4)$
- 3** **a** $(6x - 7y)(6x + 7y)$ **b** $(2x - 9y)(2x + 9y)$
 c $2(3a - 10bc)(3a + 10bc)$
- 4** **a** $(x - 1)(2x + 3)$ **b** $(3x + 1)(2x + 5)$
 c $(2x + 1)(x + 3)$ **d** $(3x - 1)(3x - 4)$
 e $(5x + 3)(2x + 3)$ **f** $2(3x - 2)(2x - 5)$
- 5** **a** $\frac{2(x+2)}{x-1}$ **b** $\frac{x}{x-1}$
 c $\frac{x+2}{x}$ **d** $\frac{x}{x+5}$
 e $\frac{x+3}{x}$ **f** $\frac{x}{x-5}$
- 6** **a** $\frac{3x+4}{x+7}$ **b** $\frac{2x+3}{3x-2}$
 c $\frac{2-5x}{2x-3}$ **d** $\frac{3x+1}{x+4}$
- 7** $(x + 5)$
- 8** $\frac{4(x+2)}{x-2}$

Rearranging equations

A LEVEL LINKS

Scheme of work: 6a. Definition, differentiating polynomials, second derivatives

Textbook: Pure Year 1, 12.1 Gradients of curves

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Examples

Example 1 Make t the subject of the formula $v = u + at$.

$v = u + at$ $v - u = at$ $t = \frac{v - u}{a}$	<ol style="list-style-type: none">1 Get the terms containing t on one side and everything else on the other side.2 Divide throughout by a.
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Example 2 Make t the subject of the formula $r = 2t - \pi t$.

$r = 2t - \pi t$ $r = t(2 - \pi)$ $t = \frac{r}{2 - \pi}$	<ol style="list-style-type: none">1 All the terms containing t are already on one side and everything else is on the other side.2 Factorise as t is a common factor.3 Divide throughout by $2 - \pi$.
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Example 3 Make t the subject of the formula $\frac{t+r}{5} = \frac{3t}{2}$.

$\frac{t+r}{5} = \frac{3t}{2}$ $2t + 2r = 15t$ $2r = 13t$ $t = \frac{2r}{13}$	<ol style="list-style-type: none">1 Remove the fractions first by multiplying throughout by 10.2 Get the terms containing t on one side and everything else on the other side and simplify.3 Divide throughout by 13.
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Example 4 Make t the subject of the formula $r = \frac{3t+5}{t-1}$.

$r = \frac{3t+5}{t-1}$ $r(t-1) = 3t+5$ $rt - r = 3t+5$ $rt - 3t = 5+r$ $t(r-3) = 5+r$ $t = \frac{5+r}{r-3}$	<ol style="list-style-type: none"> 1 Remove the fraction first by multiplying throughout by $t-1$. 2 Expand the brackets. 3 Get the terms containing t on one side and everything else on the other side. 4 Factorise the LHS as t is a common factor. 5 Divide throughout by $r-3$.
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Practice

Change the subject of each formula to the letter given in the brackets.

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|--|--|--|
| 1 $C = \pi d$ [d] | 2 $P = 2l + 2w$ [w] | 3 $D = \frac{S}{T}$ [T] |
| 4 $p = \frac{q-r}{t}$ [t] | 5 $u = at - \frac{1}{2}t$ [t] | 6 $V = ax + 4x$ [x] |
| 7 $\frac{y-7x}{2} = \frac{7-2y}{3}$ [y] | 8 $x = \frac{2a-1}{3-a}$ [a] | 9 $x = \frac{b-c}{d}$ [d] |
| 10 $h = \frac{7g-9}{2+g}$ [g] | 11 $e(9+x) = 2e+1$ [e] | 12 $y = \frac{2x+3}{4-x}$ [x] |

13 Make r the subject of the following formulae.

a $A = \pi r^2$	b $V = \frac{4}{3}\pi r^3$	c $P = \pi r + 2r$	d $V = \frac{2}{3}\pi r^2 h$
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14 Make x the subject of the following formulae.

a $\frac{xy}{z} = \frac{ab}{cd}$	b $\frac{4\pi cx}{d} = \frac{3z}{py^2}$
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15 Make $\sin B$ the subject of the formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

16 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

Extend

17 Make x the subject of the following equations.

a $\frac{p}{q}(sx+t) = x-1$	b $\frac{p}{q}(ax+2y) = \frac{3p}{q^2}(x-y)$
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Answers

$$1 \quad d = \frac{C}{\pi}$$

$$2 \quad w = \frac{P-2l}{2}$$

$$3 \quad T = \frac{S}{D}$$

$$4 \quad t = \frac{q-r}{p}$$

$$5 \quad t = \frac{2u}{2a-1}$$

$$6 \quad x = \frac{V}{a+4}$$

$$7 \quad y = 2 + 3x$$

$$8 \quad a = \frac{3x+1}{x+2}$$

$$9 \quad d = \frac{b-c}{x}$$

$$10 \quad g = \frac{2h+9}{7-h}$$

$$11 \quad e = \frac{1}{x+7}$$

$$12 \quad x = \frac{4y-3}{2+y}$$

$$13 \quad \mathbf{a} \quad r = \sqrt{\frac{A}{\pi}}$$

$$\mathbf{b} \quad r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$\mathbf{c} \quad r = \frac{P}{\pi+2}$$

$$\mathbf{d} \quad r = \sqrt{\frac{3V}{2\pi h}}$$

$$14 \quad \mathbf{a} \quad x = \frac{abz}{cdy}$$

$$\mathbf{b} \quad x = \frac{3dz}{4\pi cpy^2}$$

$$15 \quad \sin B = \frac{b \sin A}{a}$$

$$16 \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$17 \quad \mathbf{a} \quad x = \frac{q+pt}{q-ps}$$

$$\mathbf{b} \quad x = \frac{3py+2pqr}{3p-apq} = \frac{y(3+2q)}{3-aq}$$

Completing the square

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	<p>1 Write $x^2 + bx + c$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$</p> <p>2 Simplify</p>
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Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$ $= 2\left(x^2 - \frac{5}{2}x\right) + 1$ $= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$	<p>1 Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$</p> <p>2 Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$</p> <p>3 Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2</p> <p>4 Simplify</p>
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Practice

- 1 Write the following quadratic expressions in the form $(x + p)^2 + q$
- | | |
|-------------------------|--------------------------|
| a $x^2 + 4x + 3$ | b $x^2 - 10x - 3$ |
| c $x^2 - 8x$ | d $x^2 + 6x$ |
| e $x^2 - 2x + 7$ | f $x^2 + 3x - 2$ |
- 2 Write the following quadratic expressions in the form $p(x + q)^2 + r$
- | | |
|---------------------------|---------------------------|
| a $2x^2 - 8x - 16$ | b $4x^2 - 8x - 16$ |
| c $3x^2 + 12x - 9$ | d $2x^2 + 6x - 8$ |
- 3 Complete the square.
- | | |
|--------------------------|--------------------------|
| a $2x^2 + 3x + 6$ | b $3x^2 - 2x$ |
| c $5x^2 + 3x$ | d $3x^2 + 5x + 3$ |

Extend

- 4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Answers

1 a $(x+2)^2 - 1$

b $(x-5)^2 - 28$

c $(x-4)^2 - 16$

d $(x+3)^2 - 9$

e $(x-1)^2 + 6$

f $\left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$

2 a $2(x-2)^2 - 24$

b $4(x-1)^2 - 20$

c $3(x+2)^2 - 21$

d $2\left(x + \frac{3}{2}\right)^2 - \frac{25}{2}$

3 a $2\left(x + \frac{3}{4}\right)^2 + \frac{39}{8}$

b $3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3}$

c $5\left(x + \frac{3}{10}\right)^2 - \frac{9}{20}$

d $3\left(x + \frac{5}{6}\right)^2 + \frac{11}{12}$

4 $(5x+3)^2 + 3$

Solving quadratic equations by factorisation

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac .
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$ $5x(x - 3) = 0$ So $5x = 0$ or $(x - 3) = 0$ Therefore $x = 0$ or $x = 3$	<ol style="list-style-type: none">1 Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$.2 Factorise the quadratic equation. $5x$ is a common factor.3 When two values multiply to make zero, at least one of the values must be zero.4 Solve these two equations.
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Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7, ac = 12$ $x^2 + 4x + 3x + 12 = 0$ $x(x + 4) + 3(x + 4) = 0$ $(x + 4)(x + 3) = 0$ So $(x + 4) = 0$ or $(x + 3) = 0$ Therefore $x = -4$ or $x = -3$	<ol style="list-style-type: none">1 Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)2 Rewrite the b term ($7x$) using these two factors.3 Factorise the first two terms and the last two terms.4 $(x + 4)$ is a factor of both terms.5 When two values multiply to make zero, at least one of the values must be zero.6 Solve these two equations.
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Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	<ol style="list-style-type: none">1 Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$.2 When two values multiply to make zero, at least one of the values must be zero.3 Solve these two equations.
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Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5, ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\frac{3}{2}$	<ol style="list-style-type: none">1 Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3)2 Rewrite the b term ($-5x$) using these two factors.3 Factorise the first two terms and the last two terms.4 $(x - 4)$ is a factor of both terms.5 When two values multiply to make zero, at least one of the values must be zero.6 Solve these two equations.
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Practice

1 Solve

a $6x^2 + 4x = 0$

c $x^2 + 7x + 10 = 0$

e $x^2 - 3x - 4 = 0$

g $x^2 - 10x + 24 = 0$

i $x^2 + 3x - 28 = 0$

k $2x^2 - 7x - 4 = 0$

b $28x^2 - 21x = 0$

d $x^2 - 5x + 6 = 0$

f $x^2 + 3x - 10 = 0$

h $x^2 - 36 = 0$

j $x^2 - 6x + 9 = 0$

l $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$

c $x^2 + 5x = 24$

e $x(x + 2) = 2x + 25$

g $x(3x + 1) = x^2 + 15$

b $x^2 - 3 = 2x$

d $x^2 - 42 = x$

f $x^2 - 30 = 3x - 2$

h $3x(x - 1) = 2(x + 1)$

Hint

Get all terms onto one side of the equation.

Solving quadratic equations by completing the square

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$ $(x + 3)^2 - 9 + 4 = 0$ $(x + 3)^2 - 5 = 0$ $(x + 3)^2 = 5$ $x + 3 = \pm\sqrt{5}$ $x = \pm\sqrt{5} - 3$ <p>So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$</p>	<ol style="list-style-type: none"> Write $x^2 + bx + c = 0$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$ Simplify. Rearrange the equation to work out x. First, add 5 to both sides. Square root both sides. Remember that the square root of a value gives two answers. Subtract 3 from both sides to solve the equation. Write down both solutions.
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Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$ $2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$ $2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$ $2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$ $2\left(x - \frac{7}{4}\right)^2 - \frac{17}{8} = 0$	<ol style="list-style-type: none"> Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$ Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$ Expand the square brackets. Simplify. <p style="text-align: right;"><i>(continued on next page)</i></p>
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$2\left(x - \frac{7}{4}\right)^2 = \frac{17}{8}$ $\left(x - \frac{7}{4}\right)^2 = \frac{17}{16}$ $x - \frac{7}{4} = \pm \frac{\sqrt{17}}{4}$ $x = \pm \frac{\sqrt{17}}{4} + \frac{7}{4}$ <p>So $x = \frac{7}{4} - \frac{\sqrt{17}}{4}$ or $x = \frac{7}{4} + \frac{\sqrt{17}}{4}$</p>	<p>5 Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.</p> <p>6 Divide both sides by 2.</p> <p>7 Square root both sides. Remember that the square root of a value gives two answers.</p> <p>8 Add $\frac{7}{4}$ to both sides.</p> <p>9 Write down both the solutions.</p>
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Practice

3 Solve by completing the square.

a $x^2 - 4x - 3 = 0$

b $x^2 - 10x + 4 = 0$

c $x^2 + 8x - 5 = 0$

d $x^2 - 2x - 6 = 0$

e $2x^2 + 8x - 5 = 0$

f $5x^2 + 3x - 4 = 0$

4 Solve by completing the square.

a $(x - 4)(x + 2) = 5$

b $2x^2 + 6x - 7 = 0$

c $x^2 - 5x + 3 = 0$

Hint

Get all terms onto one side of the equation.

Solving quadratic equations by using the formula

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a , b and c .

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$$a = 1, b = 6, c = 4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{20}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{5}}{2}$$

$$x = -3 \pm \sqrt{5}$$

$$\text{So } x = -3 - \sqrt{5} \text{ or } x = \sqrt{5} - 3$$

- 1 Identify a , b and c and write down the formula.

Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.

- 2 Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
- 3 Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
- 4 Simplify $\sqrt{20}$.
 $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
- 5 Simplify by dividing numerator and denominator by 2.
- 6 Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$ $x = \frac{7 \pm \sqrt{73}}{6}$ <p>So $x = \frac{7 - \sqrt{73}}{6}$ or $x = \frac{7 + \sqrt{73}}{6}$</p>	<ol style="list-style-type: none">1 Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.2 Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.3 Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.4 Write down both the solutions.
---	---

Practice

5 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$

b $2x^2 - 4x - 7 = 0$

6 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a , b and c are integers.

7 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

Hint

Get all terms onto one side of the equation.

Extend

8 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

Answers

- 1**
- a** $x = 0$ or $x = -\frac{2}{3}$
- b** $x = 0$ or $x = \frac{3}{4}$
- c** $x = -5$ or $x = -2$
- d** $x = 2$ or $x = 3$
- e** $x = -1$ or $x = 4$
- f** $x = -5$ or $x = 2$
- g** $x = 4$ or $x = 6$
- h** $x = -6$ or $x = 6$
- i** $x = -7$ or $x = 4$
- j** $x = 3$
- k** $x = -\frac{1}{2}$ or $x = 4$
- l** $x = -\frac{2}{3}$ or $x = 5$
- 2**
- a** $x = -2$ or $x = 5$
- b** $x = -1$ or $x = 3$
- c** $x = -8$ or $x = 3$
- d** $x = -6$ or $x = 7$
- e** $x = -5$ or $x = 5$
- f** $x = -4$ or $x = 7$
- g** $x = -3$ or $x = 2\frac{1}{2}$
- h** $x = -\frac{1}{3}$ or $x = 2$
- 3**
- a** $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$
- b** $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$
- c** $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$
- d** $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$
- e** $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$
- f** $x = \frac{-3 + \sqrt{89}}{10}$ or $x = \frac{-3 - \sqrt{89}}{10}$
- 4**
- a** $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$
- b** $x = \frac{-3 + \sqrt{23}}{2}$ or $x = \frac{-3 - \sqrt{23}}{2}$
- c** $x = \frac{5 + \sqrt{13}}{2}$ or $x = \frac{5 - \sqrt{13}}{2}$
- 5**
- a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$
- b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$
- 6** $x = \frac{7 + \sqrt{41}}{2}$ or $x = \frac{7 - \sqrt{41}}{2}$
- 7** $x = \frac{-3 + \sqrt{89}}{20}$ or $x = \frac{-3 - \sqrt{89}}{20}$
- 8**
- a** $x = \frac{7 + \sqrt{17}}{8}$ or $x = \frac{7 - \sqrt{17}}{8}$
- b** $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$
- c** $x = -1\frac{2}{3}$ or $x = 2$

Solving linear simultaneous equations using the elimination method

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$\begin{array}{r} 3x + y = 5 \\ - \quad x + y = 1 \\ \hline 2x \quad = 4 \\ \text{So } x = 2 \end{array}$	1 Subtract the second equation from the first equation to eliminate the y term.
$\begin{array}{l} \text{Using } x + y = 1 \\ \quad 2 + y = 1 \\ \text{So } y = -1 \end{array}$	2 To find the value of y , substitute $x = 2$ into one of the original equations.
$\begin{array}{l} \text{Check:} \\ \text{equation 1: } 3 \times 2 + (-1) = 5 \quad \text{YES} \\ \text{equation 2: } 2 + (-1) = 1 \quad \text{YES} \end{array}$	3 Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$\begin{array}{r} x + 2y = 13 \\ + \quad 5x - 2y = 5 \\ \hline 6x \quad = 18 \\ \text{So } x = 3 \end{array}$	1 Add the two equations together to eliminate the y term.
$\begin{array}{l} \text{Using } x + 2y = 13 \\ \quad 3 + 2y = 13 \\ \text{So } y = 5 \end{array}$	2 To find the value of y , substitute $x = 3$ into one of the original equations.
$\begin{array}{l} \text{Check:} \\ \text{equation 1: } 3 + 2 \times 5 = 13 \quad \text{YES} \\ \text{equation 2: } 5 \times 3 - 2 \times 5 = 5 \quad \text{YES} \end{array}$	3 Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$\begin{array}{r} (2x + 3y = 2) \times 4 \rightarrow 8x + 12y = 8 \\ (5x + 4y = 12) \times 3 \rightarrow 15x + 12y = 36 \\ \hline 7x = 28 \end{array}$	<p>1 Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.</p> <p>2 To find the value of y, substitute $x = 4$ into one of the original equations.</p> <p>3 Substitute the values of x and y into both equations to check your answers.</p>
<p>So $x = 4$</p> <p>Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$</p> <p>Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES</p>	

Practice

Solve these simultaneous equations.

1 $4x + y = 8$
 $x + y = 5$

2 $3x + y = 7$
 $3x + 2y = 5$

3 $4x + y = 3$
 $3x - y = 11$

4 $3x + 4y = 7$
 $x - 4y = 5$

5 $2x + y = 11$
 $x - 3y = 9$

6 $2x + 3y = 11$
 $3x + 2y = 4$

Solving linear simultaneous equations using the substitution method

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Textbook: Pure Year 1, 3.1 Linear simultaneous equations

Key points

- The substitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$ $5x + 6x + 3 = 14$ $11x + 3 = 14$ $11x = 11$ $\text{So } x = 1$ $\text{Using } y = 2x + 1$ $y = 2 \times 1 + 1$ $\text{So } y = 3$ Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	<ol style="list-style-type: none">1 Substitute $2x + 1$ for y into the second equation.2 Expand the brackets and simplify.3 Work out the value of x.4 To find the value of y, substitute $x = 1$ into one of the original equations.5 Substitute the values of x and y into both equations to check your answers.
--	--

Example 5 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$ $4x + 3(2x - 16) = -3$ $4x + 6x - 48 = -3$ $10x - 48 = -3$ $10x = 45$ $\text{So } x = 4\frac{1}{2}$ $\text{Using } y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ $\text{So } y = -7$ Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	<ol style="list-style-type: none">1 Rearrange the first equation.2 Substitute $2x - 16$ for y into the second equation.3 Expand the brackets and simplify.4 Work out the value of x.5 To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations.6 Substitute the values of x and y into both equations to check your answers.
--	--

Practice

Solve these simultaneous equations.

7 $y = x - 4$
 $2x + 5y = 43$

8 $y = 2x - 3$
 $5x - 3y = 11$

9 $2y = 4x + 5$
 $9x + 5y = 22$

10 $2x = y - 2$
 $8x - 5y = -11$

11 $3x + 4y = 8$
 $2x - y = -13$

12 $3y = 4x - 7$
 $2y = 3x - 4$

13 $3x = y - 1$
 $2y - 2x = 3$

14 $3x + 2y + 1 = 0$
 $4y = 8 - x$

Extend

15 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \frac{3(y - x)}{4}$.

Answers

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$

6 $x = -2, y = 5$

7 $x = 9, y = 5$

8 $x = -2, y = -7$

9 $x = \frac{1}{2}, y = 3\frac{1}{2}$

10 $x = \frac{1}{2}, y = 3$

11 $x = -4, y = 5$

12 $x = -2, y = -5$

13 $x = \frac{1}{4}, y = 1\frac{3}{4}$

14 $x = -2, y = 2\frac{1}{2}$

15 $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

Straight line graphs

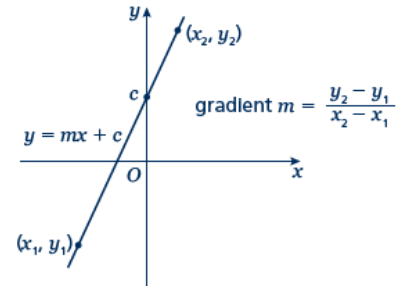
A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y -intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a , b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the

$$\text{formula } m = \frac{y_2 - y_1}{x_2 - x_1}$$



Examples

Example 1 A straight line has gradient $-\frac{1}{2}$ and y -intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2} \text{ and } c = 3$ $\text{So } y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	<ol style="list-style-type: none"> 1 A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. 2 Rearrange the equation so all the terms are on one side and 0 is on the other side. 3 Multiply both sides by 2 to eliminate the denominator.
--	--

Example 2 Find the gradient and the y -intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ $\text{Gradient} = m = \frac{2}{3}$ $\text{y-intercept} = c = -\frac{4}{3}$	<ol style="list-style-type: none"> 1 Make y the subject of the equation. 2 Divide all the terms by three to get the equation in the form $y = \dots$ 3 In the form $y = mx + c$, the gradient is m and the y-intercept is c.
--	--

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$ $13 = 3 \times 5 + c$ $13 = 15 + c$ $c = -2$ $y = 3x - 2$	<ol style="list-style-type: none"> 1 Substitute the gradient given in the question into the equation of a straight line $y = mx + c$. 2 Substitute the coordinates $x = 5$ and $y = 13$ into the equation. 3 Simplify and solve the equation. 4 Substitute $c = -2$ into the equation $y = 3x + c$
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Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2, x_2 = 8, y_1 = 4 \text{ and } y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 4}{8 - 2} = \frac{3}{6} = \frac{1}{2}$ $y = \frac{1}{2}x + c$ $4 = \frac{1}{2} \times 2 + c$ $c = 3$ $y = \frac{1}{2}x + 3$	<ol style="list-style-type: none"> 1 Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line. 2 Substitute the gradient into the equation of a straight line $y = mx + c$. 3 Substitute the coordinates of either point into the equation. 4 Simplify and solve the equation. 5 Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$
---	---

Practice

1 Find the gradient and the y-intercept of the following equations.

- | | |
|----------------------------|----------------------------------|
| a $y = 3x + 5$ | b $y = -\frac{1}{2}x - 7$ |
| c $2y = 4x - 3$ | d $x + y = 5$ |
| e $2x - 3y - 7 = 0$ | f $5x + y - 4 = 0$ |

Hint
Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	y-intercept	Equation of the line
5	0	
-3	2	
4	-7	

- 3** Find, in the form $ax + by + c = 0$ where a , b and c are integers, an equation for each of the lines with the following gradients and y -intercepts.
- a** gradient $-\frac{1}{2}$, y -intercept -7 **b** gradient 2 , y -intercept 0
- c** gradient $\frac{2}{3}$, y -intercept 4 **d** gradient -1.2 , y -intercept -2
- 4** Write an equation for the line which passes through the point $(2, 5)$ and has gradient 4 .
- 5** Write an equation for the line which passes through the point $(6, 3)$ and has gradient $-\frac{2}{3}$.
- 6** Write an equation for the line passing through each of the following pairs of points.
- a** $(4, 5)$, $(10, 17)$ **b** $(0, 6)$, $(-4, 8)$
- c** $(-1, -7)$, $(5, 23)$ **d** $(3, 10)$, $(4, 7)$

Extend

- 7** The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

Answers

- 1** **a** $m = 3, c = 5$ **b** $m = -\frac{1}{2}, c = -7$
 c $m = 2, c = -\frac{3}{2}$ **d** $m = -1, c = 5$
 e $m = \frac{2}{3}, c = -\frac{7}{3}$ or $-2\frac{1}{3}$ **f** $m = -5, c = 4$

2

Gradient	y-intercept	Equation of the line
5	0	$y = 5x$
-3	2	$y = -3x + 2$
4	-7	$y = 4x - 7$

- 3** **a** $x + 2y + 14 = 0$ **b** $2x - y = 0$
 c $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

4 $y = 4x - 3$

5 $y = -\frac{2}{3}x + 7$

6 **a** $y = 2x - 3$ **b** $y = -\frac{1}{2}x + 6$

c $y = 5x - 2$ **d** $y = -3x + 19$

7 $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the y-intercept is 3.

The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $(4, -3)$.

Trigonometry: The cosine rule

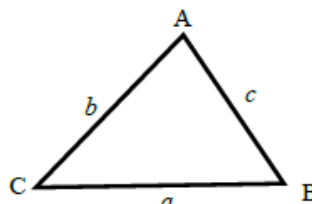
A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

Textbook: Pure Year 1, 9.1 The cosine rule

Key points

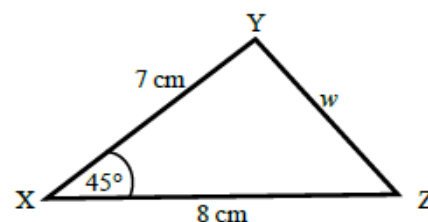
- a is the side opposite angle A .
- b is the side opposite angle B .
- c is the side opposite angle C .

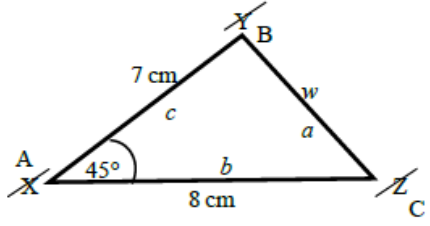


- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc \cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

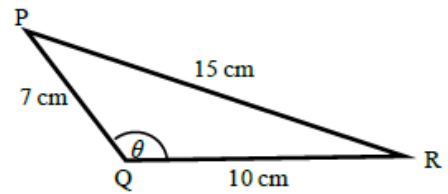
Examples

Example 4 Work out the length of side w .
Give your answer correct to 3 significant figures.



 $a^2 = b^2 + c^2 - 2bc \cos A$ $w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45^\circ$ $w^2 = 33.804\ 040\ 51\dots$ $w = \sqrt{33.804\ 040\ 51}$ $w = 5.81 \text{ cm}$	<ol style="list-style-type: none">1 Always start by labelling the angles and sides.2 Write the cosine rule to find the side.3 Substitute the values a, b and A into the formula.4 Use a calculator to find w^2 and then w.5 Round your final answer to 3 significant figures and write the units in your answer.
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Example 5 Work out the size of angle θ .
Give your answer correct to 1 decimal place.

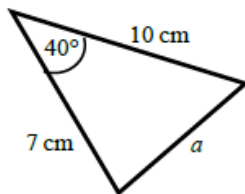


<p> $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos \theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$ $\cos \theta = \frac{-76}{140}$ $\theta = 122.878\ 349\dots$ $\theta = 122.9^\circ$ </p>	<ol style="list-style-type: none"> 1 Always start by labelling the angles and sides. 2 Write the cosine rule to find the angle. 3 Substitute the values a, b and c into the formula. 4 Use \cos^{-1} to find the angle. 5 Use your calculator to work out $\cos^{-1}(-76 \div 140)$. 6 Round your answer to 1 decimal place and write the units in your answer.
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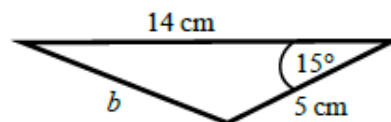
Practice

6 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

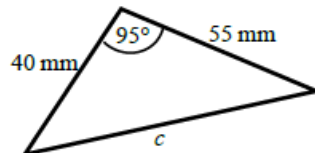
a



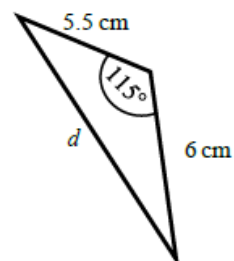
b



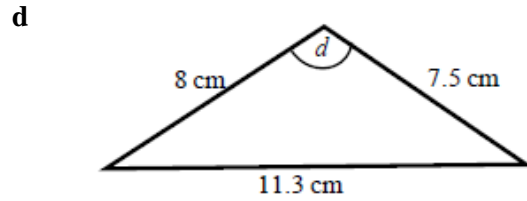
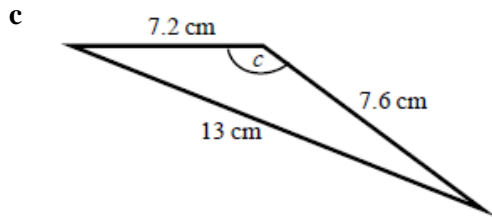
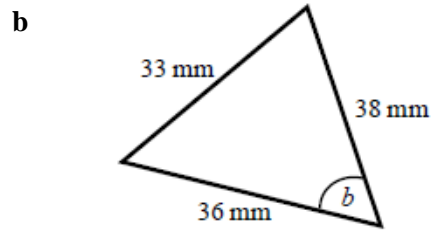
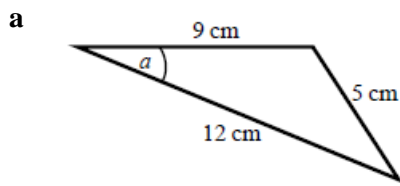
c



d

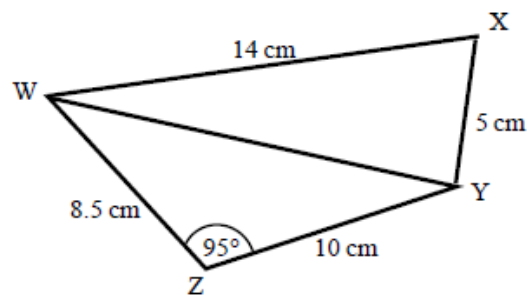


7 Calculate the angles labelled θ in each triangle. Give your answer correct to 1 decimal place.



8 a Work out the length of WY. Give your answer correct to 3 significant figures.

b Work out the size of angle WXY. Give your answer correct to 1 decimal place.



Trigonometry: The sine rule

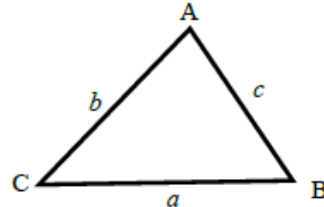
A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

Textbook: Pure Year 1, 9.2 The sine rule

Key points

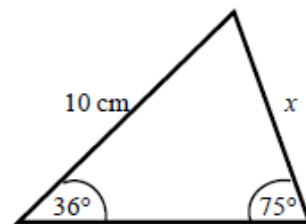
- a is the side opposite angle A .
- b is the side opposite angle B .
- c is the side opposite angle C .



- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

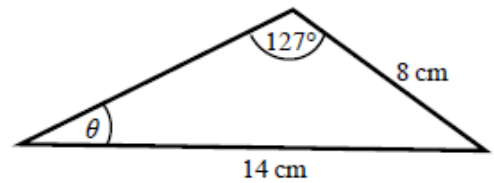
Examples

Example 6 Work out the length of side x .
Give your answer correct to 3 significant figures.



$\frac{a}{\sin A} = \frac{b}{\sin B}$ $\frac{x}{\sin 36^\circ} = \frac{10}{\sin 75^\circ}$ $x = \frac{10 \times \sin 36^\circ}{\sin 75^\circ}$ $x = 6.09 \text{ cm}$	<ol style="list-style-type: none">1 Always start by labelling the angles and sides.2 Write the sine rule to find the side.3 Substitute the values a, b, A and B into the formula.4 Rearrange to make x the subject.5 Round your answer to 3 significant figures and write the units in your answer.
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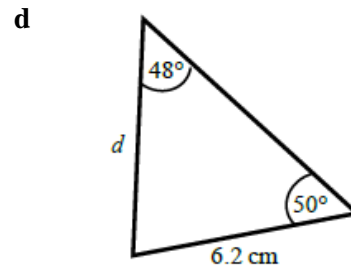
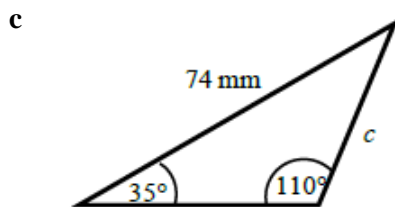
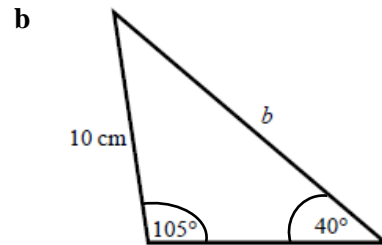
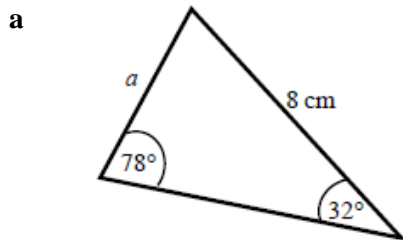
Example 7 Work out the size of angle θ .
Give your answer correct to 1 decimal place.



$\frac{\sin A}{a} = \frac{\sin B}{b}$ $\frac{\sin \theta}{8} = \frac{\sin 127^\circ}{14}$ $\sin \theta = \frac{8 \times \sin 127^\circ}{14}$ $\theta = 27.2^\circ$	<ol style="list-style-type: none"> 1 Always start by labelling the angles and sides. 2 Write the sine rule to find the angle. 3 Substitute the values a, b, A and B into the formula. 4 Rearrange to make $\sin \theta$ the subject. 5 Use \sin^{-1} to find the angle. Round your answer to 1 decimal place and write the units in your answer.
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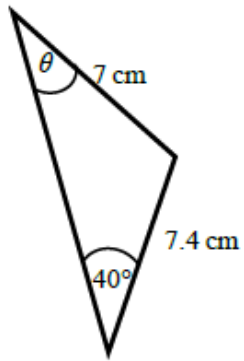
Practice

9 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

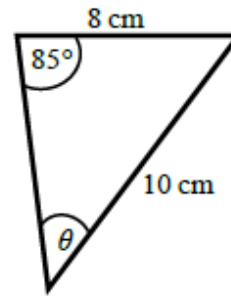


- 10 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

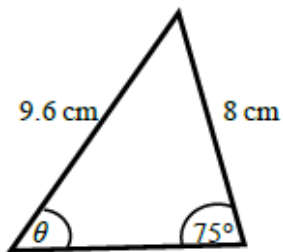
a



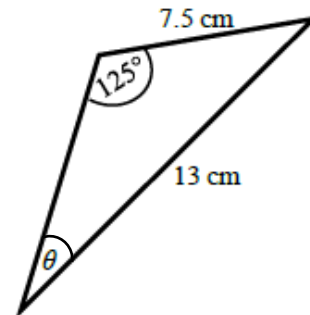
b



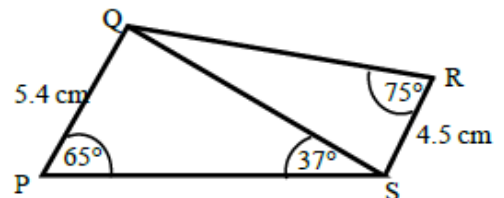
c



d



- 11 a Work out the length of QS.
Give your answer correct to 3 significant figures.
- b Work out the size of angle RQS.
Give your answer correct to 1 decimal place.



Areas of non-right angled triangles

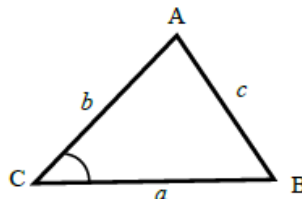
A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

Textbook: Pure Year 1, 9.3 Areas of triangles

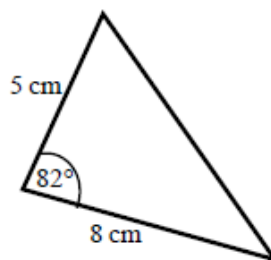
Key points

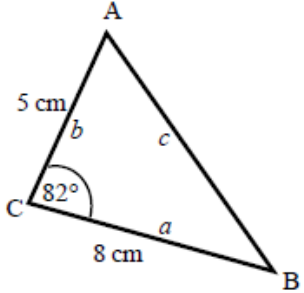
- a is the side opposite angle A .
 b is the side opposite angle B .
 c is the side opposite angle C .
- The area of the triangle is $\frac{1}{2}ab \sin C$.



Examples

Example 8 Find the area of the triangle.

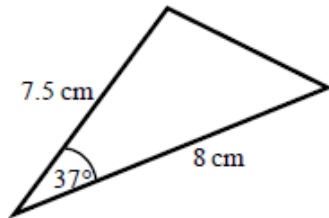


 <p>Area = $\frac{1}{2}ab \sin C$</p> <p>Area = $\frac{1}{2} \times 8 \times 5 \times \sin 82^\circ$</p> <p>Area = 19.805 361...</p> <p>Area = 19.8 cm²</p>	<ol style="list-style-type: none">1 Always start by labelling the sides and angles of the triangle.2 State the formula for the area of a triangle.3 Substitute the values of a, b and C into the formula for the area of a triangle.4 Use a calculator to find the area.5 Round your answer to 3 significant figures and write the units in your answer.
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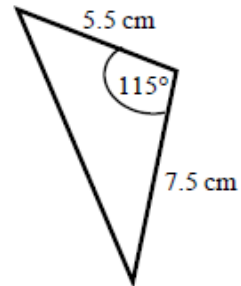
Practice

- 12 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

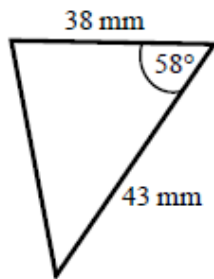
a



b



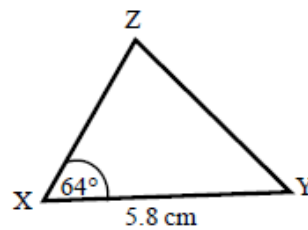
c



- 13 The area of triangle XYZ is 13.3 cm^2 .
Work out the length of XZ.

Hint:

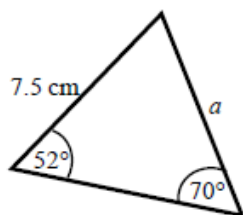
Rearrange the formula to make a side the subject.



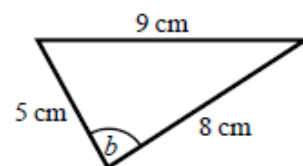
Extend

- 14 Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

a



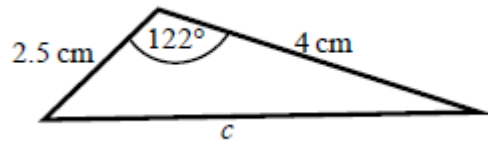
b



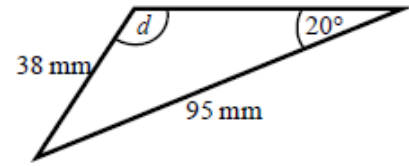
Hint:

For each one, decide whether to use the cosine or sine rule.

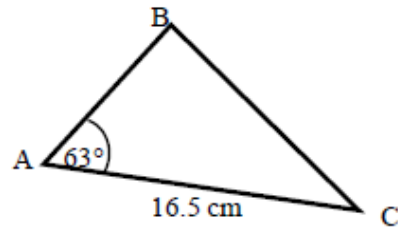
c



d



- 15 The area of triangle ABC is 86.7 cm^2 .
Work out the length of BC.
Give your answer correct to 3 significant figures.



Answers

6 **a** 6.46 cm **b** 9.26 cm **c** 70.8 mm **d** 9.70 cm

7 **a** 22.2° **b** 52.9° **c** 122.9° **d** 93.6°

8 **a** 13.7 cm **b** 76.0°

9 **a** 4.33 cm **b** 15.0 cm **c** 45.2 mm **d** 6.39 cm

10 **a** 42.8° **b** 52.8° **c** 53.6° **d** 28.2°

11 **a** 8.13 cm **b** 32.3°

12 **a** 18.1 cm² **b** 18.7 cm² **c** 693 mm²

13 5.10 cm

14 **a** 6.29 cm **b** 84.3° **c** 5.73 cm **d** 58.8°

15 15.3 cm